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Saint-Venant torsion of orthotropic bars with rectangular cross section weakened by cracks

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ABSTRACT

The solution to problem of a Volterra-type screw dislocation in an orthotropic bar with rectangular cross section is first obtained by means of a finite Fourier cosine transform. The bar is under axial torque which is governed by the Saint-Venant torsion theory. The series solution is then derived for displacement and stress fields in the bar cross section. The dislocation solution is employed to derive a set of Cauchy singular integral equations for the analysis of curved cracks. The solution to these equations is used to determine the torsional rigidity of bar and the stress intensity factors (SIFs) for the tips of the cracks. Several examples of a single straight crack and an arc-crack are solved. Furthermore, the interaction between two cracks is studied. Finally, the stress components around an inclined edge crack tip are used to define the boundary of the plastic region employing von Mises yield criterion.

1. Introduction

Shafts or bars in torsion are often subjected to cracking during their service life. Though the torsion problem of bars is a rather old one in the theory of elasticity, the torsion problem of bars with multiple curved cracks has not been adequately developed. The problems of an elastic bar in torsion have been considered by a number of investigators. These investigations may be grouped into two major categories: those primarily dealing with bars with no defects, and those studying bars with single or multiple defects. Within the first category, there are numerous investigations in the literature (Ecsedi, 2009; Rongqiao et al., 2010).The bars contained with multiple defects have been the subject of other earlier investigations. First, bars with rectangular cross section are reviewed.

Torsion problem of a rectangular bar containing one or two edge cracks perpendicular to the cross section sides was developed by Chen (1980). The succeeding Dirichlet problem of the Laplace equation was solved by subdividing the cross section into several rectangular regions and using the Duhem theorem (a theorem of continuation of harmonic function). The torsional rigidity and also the compliance coefficient were obtained and lastly, the stress intensity factors of crack tips were calculated. The applied method was restricted only to the edge cracks, which were normal to the sides of cross section.

Chen et al. (1997) gave a solution of the torsion problem for an orthotropic rectangular bar containing an edge crack, bisecting and normal to one side of the cross section. The solution of governing equation was accomplished by mapping a rectangular plane with cut to another one. Indeed, by applying this mapping, the problem was reduced to the Laplace equation. After solving this Laplace equation, the compliance coefficient as an inverse of torsion rigidity coefficient was evaluated. Considering the fact that energy release rate can be obtained in terms of applied torque and the compliance coefficient and also as a function of stress intensity factor of crack tip, a formula for the stress intensity factor was presented.

The flexural and torsional problem of bars with circular, elliptical and semi-elliptical cross sections weakened by some edge and embedded cracks was subject of study by Sih (1963). The problem was formulated based on three complex flexure functions including the classical torsion function and the non-trivial stresses were calculated in terms of the aforementioned complex functions. By choosing the appropriate complex flexure function, which satisfies the necessary boundary conditions of outer boundary of the bar and also those corresponding to crack, the singular stress field near the crack tips was found. Then, the closed form relations for stress intensity factors of crack tips were given.

The analytical solutions for the torsion rigidities of a circular bar weakened by different length radial cracks were achieved by Lebedev et al. (1979). Also, the problem of the twisting of a rod of elliptical cross section with two edge cracks extending to its foci was analyzed and the torsional rigidity was calculated.

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Chen and Chen (1983) conducted the solution of a bar with a cracked ring section under torsion. The cross section weakened by three edge cracks emanating from outer boundary of section and all the cracks were equally spaced. By partitioning the plan of cross section and applying conformal mapping, the problem was analyzed in the rectangular region and by employing the harmonic function continuation technique the final solution was derived. Finally, torsional rigidity and stress intensity factors of tips of cracks were achieved.

Duality between the problem of the circular plane with multiple cracks in anti-plane elasticity and problem of multiple cracks of the round bar under torsion was proposed by Chen (1984). Namely, one should interprets the warping function of bar as the displacement function in anti-plane elasticity. The method of complex functions was used for analyzing the problem.

Using the Hankel transform and Fourier series, the torsion problem of a finite cylinder containing a concentric penny-shaped crack was solved by Chang (1985). The governing equation of the problem was in terms of the only non-zero displacement component, namely the circumferential component of displacement. The problem was reduced to a Fredholm integral equation of the second kind which was solved by an iterative scheme and the Mode III stress intensity factor of the crack tip was evaluated.

Torsion of a finite orthotropic cylinder with a concentric penny-shaped crack was treated by Zhang (1988). Making use of the Hankel transform and Fourier series, the problem was reduced to a Fredholm integral equation of the second kind. The numerical solution of this equation led to stress intensity factor of the crack tip.

Analytical expression of the torsion function for a cylinder containing a screw dislocation was first given in the work accomplished by Xiao-chun and Ren-ji (1988). This solution may also be found by the method of images. Using the corresponding dislocation solution and via dislocation distribution technique, the problem was reduced to solve a system of singular integral equations for the unknown dislocation density functions. Numerical solution of the ensuing singular integral equations was led to the torsional rigidity of cylinder and also stress intensity factors of crack tips for a cylinder with several cracks was obtained.

Renji and Yulan (1992) presented a set of solutions for torsional problems of a cylinder with a rectangular hole and a rectangular cylinder containing a crack. The torsion problem of circular cylinder containing several cracks solved by dislocation distribution technique was split into two above-mentioned problems. Then, torsional rigidities and the stress intensity factor at the crack tip were determined. Also for the circular cylinder with a rectangular hole the expressions for the singular stresses around the concave corner points were derived and the stress concentration factors were achieved.

Yi-Zhou and Yi-Heng (1983) focused on the study of a bar with sector shape cross section under torsion, weakened by radial and circumferential edge cracks. Using the conformal mapping the cross section was transformed to a rectangular cross section with edge cracks normal to the sides of cross section. Subsequently, analogous to reference Chen (1999), a general solution of the Dirichlet problem for the Laplace equation in the bar with rectangular cross section was obtained by using the harmonic function continuation technique as well as the compliance method. Finally, stress intensity factors of crack tips were evaluated.

Fang-ming and Ren-ji (1993) analyzed Saint-Venant torsion of a circular cylinder with internal crack that reinforced by an eccentric rod with different material of cylinder. Using Muskhelishvili single-layer potential function solution and single crack solution for the torsion problem of a cylinder, the problem was reduced to a set of mixed-type integral equation with generalized Cauchy-kernel. These integral equations were a mixture of Fredholm integrals of single-layer potential density functions and also Cauchy-type singular integrals of unknown dislocation density function. By numerical solution of these integral equations, torsional rigidity and stress intensity factors were obtained.

There is a subgroup of torsion problems dealing with bars weakened by circumferential and interfacial defects which are reviewed in the following.

Problem of torsion and extension of an infinite solid cylinder with an edge annular crack as a slit on the outer surface of cylinder was treated by Kudriavtsev and Parton (1973). The stress field was determined in the vicinity of slit which its surfaces were normal to the axis of cylinder and the crack was symmetric with respect to the cylinder axis. In the case of pure torsion and extension along the cylinder axis, they found stress intensity factors of the crack tips via the stress field.

Torsion of a long bar with circular cross-section and uniform surface coating layer subjected to external torque at two ends was investigated by Wu et al. (2008). In their study a periodic of circumferential cracks in the coating layer governed by pure shearing was considered. To determine the progressive cracking density in the coating layers, the free-edge stresses near crack tip and the shear stresses in the cross-section of bar were obtained based on the variational principle of complementary strain energy. Assuming strain energy conservation, a criterion for progressive cracking in the coating layer was established and the crack density was estimated.

Problem of torsion of a cylindrical interface crack in a bi-layered tubular composite of finite thickness was investigated by Li et al. (2013). Two elastic cylinders were perfectly bonded to each other on the cylindrical surfaces, except for an interface cylindrical crack. The problem was assumed to be axisymmetric and by applying cosine integral transform to the governing equation, the problem was led to a Cauchy singular integral equation which was solved numerically to obtain stress intensity factor of crack tips. The simultaneous effect of geometrical and physical parameters on the interfacial fracture behavior was discussed.

Lazzarin et al. (2007), were derived the closed form stress fields of a semi-elliptic circumferential notch and peripheral crack in an axisymmetric shaft under torsional loading. The problem was formulated by using complex potential functions and choosing a proper elliptic coordinate system having the origin located at the center of the notch. Solution of governing Laplace's equation was written as the real part of an analytical (holomorphic) function and stress field was attained in terms of complex potential function. By choosing a suitable complex potential function, the boundary conditions of the problem were satisfied and the stress field was acquired.

Relations for the Mode III notch stress intensity factors, NSIFs, for circumferentially-sharply-notched rounded bars under torsion were presented by Zappalorto et al. (2009). A closed-form solution for the NSIFs was obtained for deep notches; subsequently the solution was extended also to finite notched components.

In the last part of this review, we cite investigations with emphasis on numerical methods.

The torsion problem of a rectangular cross section bar with inner crack as two Dirichlet problems with the definite boundary values was studied by Chen (1998). The corresponding Dirichlet problems are solved using the finite difference method. The torsion rigidity coefficient and the subsequent compliance coefficient as an inverse of the torsion rigidity coefficient was reached by numerical integration. At the end, the stress intensity factor for an embedded straight crack was evaluated. Also, the evaluation was carried out for four inner branch cracks, which are perpendicular to each other.

In another study by Chen (1999), the torsion problem of thin-walled cylinder weakened by four inner edge crack normal to inner