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Cohesive-length scales for damage and toughening mechanisms

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ABSTRACT

While toughening and damage might seem to be two contradictory concepts for the mechanics of crack growth, they are actually the same phenomena perceived from two different vantage points. Similarly, the concepts of extrinsic and intrinsic toughening, defined in terms of whether a toughening mechanism occurs behind or ahead of a crack, depend on the definition of a crack tip that, in the absence of a singularity, can be somewhat arbitrary. Cohesive-zone models provide useful numerical tools for rationalizing these different concepts and, here, we use them to show how different perspectives of toughening and damage can be understood.

The concept of a cohesive length, defined in terms of an effective modulus and the magnitudes of the local tractions and displacements (or work done), can be generalized so that it can be used at any load before failure, and at any point along the interface. We show that this general concept allows multiple damage and toughening mechanisms, each with its own characteristic cohesive length, to be described and tracked in terms of a single traction–separation law. In general, the onset of damage corresponds to an increase in cohesive length. This tends to weaken a material unless compensated for by a sufficiently high increment of additional toughness. The ratio between the cohesive length of a particular damage/toughening mechanism and any relevant geometrical length determines whether the mechanism needs to be included in the cohesive-zone formulation. Furthermore, it appears that diffuse damage and crack jumping between interfaces may be induced when the cohesive length of a damage mechanism is large compared to a micro-structural length. It is speculated that this may be of some relevance to the design of hierarchical materials.

1. Introduction

Toughening mechanisms can be categorized into two types: *intrinsic* and *extrinsic* (Fig. 1) (Ritchie, 2011). Intrinsic mechanisms are the dissipative processes that occur ahead of the crack tip in what is known as a crack-tip process zone. Extrinsic mechanisms are those that occur behind the crack tip. Examples of intrinsic toughening include plasticity, void growth, micro-cracking, phase changes and crazing. Examples of extrinsic toughening include bridging zones and the unloading of a crack-tip process zone as it passes into the wake of a crack. However, as will be emphasized in this work, the physical reality that singular stresses do not occur in real materials means that the location of a crack tip can be arbitrarily defined, and the division of toughening mechanisms into intrinsic and extrinsic is a matter of perspective and convenience. For example, the crack tip could be defined as being the point at which there is no interaction between the crack surfaces. In this case, all deformation up to the point of rupture would be associated with intrinsic toughening. Alternatively, the crack tip could be defined as being the point at which one deformation process ceases; for example, the point at which the matrix material ruptures leaving bridging fibers as the only interaction between the crack surfaces. Any mechanism acting ahead of this point would then be associated with intrinsic toughening; any mechanism acting behind would be associated with extrinsic toughening.

Recognition of the arbitrary nature of the definition of a crack tip is important because understanding the processes by which one might strengthen a material can be influenced by one's perception of the nature of the toughening. Furthermore, it should be recognized that intrinsic toughening mechanisms that introduce an aspect of non-linearity into the crack-tip process zone ahead of a crack could equally-well be viewed as damage mechanisms that might be perceived to weaken a material (Thouless, 1988). Indeed, the question of whether damage (intrinsic-toughening)

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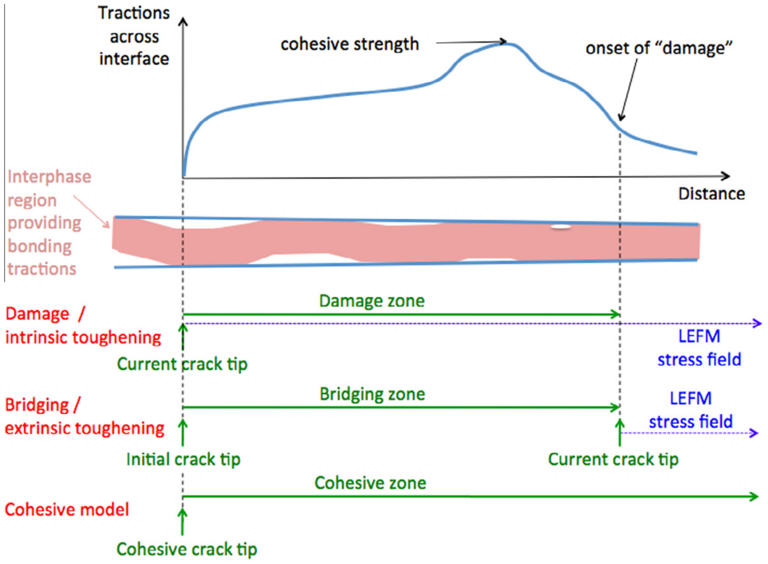

Fig. 1. Comparisons between the assumed locations of crack tips for extrinsic toughening models, intrinsic toughening models and cohesive-zone models. All of these represent the same physical reality.

mechanisms strengthen or weaken a material is addressed in this paper.

Cohesive-zone models provide useful analytical tools for exploring the concepts of toughening and damage (Fig. 1). Traction–separation laws, which dictate the tractions across a crack plane as a function of separation distance, can be used as a means of representing various forms of crack-tip processes into a finite-element analysis that allows crack propagation to evolve naturally upon loading. Numerical experiments can be performed in which the stress evolution, crack propagation and applied loads are investigated for different forms of cohesive laws, and the results can be interpreted from different perspectives of toughening, and with different definitions of the crack tip. Since the behavior of the body from a global perspective has to be independent of any perspective chosen to describe the mechanics (Fig. 1), this approach provides a means to rationalize different perceptions of toughening under unifying concepts.

The concept of a cohesive, fracture, or bridging length has been established for composites and other materials (Hillerborg et al., 1976; Bao and Suo, 1992). For mode-I, this length is dependent on three important parameters: (i) the cohesive or bridging strength, $\hat{\sigma}$, which is the maximum stress that can be supported by any element of material in the crack plane; (ii) the mode-I toughness, Γ_I, which is the total energy dissipated by creating unit area of new crack surface, including fracture of any ligaments across the crack plane; and (iii) the effective modulus of the material on either side of the interface, $\bar{E}^*$. These three quantities can be combined to give a material parameter with dimensions of length, so that a nominal mode-I fracture length can be defined as (Hillerborg et al., 1976; Bao and Suo, 1992; Sills and Thouless, 2013)

$$\zeta_I = \frac{\bar{E}^*\Gamma_I}{\hat{\sigma}^2}, \tag{1a}$$

where the effective modulus for a bi-material system is

$$\bar{E}^* = \frac{2\bar{E}_1\bar{E}_2}{\bar{E}_1 + \bar{E}_2}, \tag{1b}$$

$\bar{E}$ is Young's modulus (E in plane stress, $E/(1-\nu^2)$ in plane strain, and ν is Poisson's ratio), and the subscripts 1 and 2 refer to the materials on either side of the interface.

The ratio of the nominal fracture length to the smallest geometrical dimension associated with fracture, such as the crack length, a, the uncracked ligament length, L, or the laminate thickness, h, gives a non-dimensional nominal fracture-length scale. If this fracture-length scale is small, less than about 0.4,[1] crack growth is controlled by the toughness. An energy-release rate based on linear-elasticity can be calculated and compared to the interfacial toughness to determine if the crack will grow. As will be discussed later, this gives an upper-bound for the strength of a bonded system. If the fracture-length scale is large, greater than about 2, crack growth is controlled by the cohesive strength (Parmigiani and Thouless, 2007). This provides a second upper-bound for the strength of a bonded system that can be obtained by equating the average stress supported by the interface to its cohesive strength. At intermediate scales, there is a smooth transition between toughness- and strength-controlled fracture. The two limits are linked to the concepts of notch sensitivity and insensitivity (Bao and Suo, 1992). When failure is controlled by toughness, the strength of the material is sensitive to geometrical stress concentrations, such as cracks or changes in section. When failure is controlled by cohesive strength, geometrical features do not concentrate stresses, leaving the strength to be dictated by the average stress on the smallest load-bearing ligament.In a companion paper (Sills and Thouless, 2013), it was observed that an instantaneous cohesive length, defined in terms of the displacement and work done by the cohesive tractions, can be defined for any increment of loading up to, and including, the point of fracture. The definition of the instantaneous cohesive length can be illustrated by reference to a generic form of the mode-I traction–separation law that will be used in this paper (Fig. 2). When the instantaneous displacement from the equilibrium separation of the interface is δ_n, the cohesive tractions have done work $\mathcal{W}_I$. The mode-I instantaneous cohesive length is then defined as

$$\xi_I = \frac{\bar{E}^*\delta_n^2}{\mathcal{W}_I} = \frac{\bar{E}^*\mathcal{W}_I}{\sigma_{avg}^2}, \tag{2}$$

where σ_{avg} is the average stress exerted by the cohesive element up to the displacement of interest. An instantaneous cohesive-length

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[1] This value of 0.4 has been selected to make a connection to the common definition for the validity of linear-elastic fracture mechanics.