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Three-dimensional invariant-based failure criteria for fibre-reinforced composites

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A new three-dimensional failure criteria for fibre-reinforced composite materials based on structural tensors is presented. The transverse and the longitudinal failure mechanisms are formulated considering different criteria: for the transverse failure mechanisms, the proposed three-dimensional invariant-based criterion is formulated directly from invariant theory. Regarding the criteria for longitudinal failure, tensile fracture in the fibre direction is predicted using a noninteracting maximum allowable strain criterion. Longitudinal compressive failure is predicted using a three-dimensional kinking model based on the invariant failure criteria formulated for transverse fracture. The proposed kinking model is able to account for the nonlinear shear response typically observed in fibre-reinforced polymers. For validation, the failure envelopes for several fibre-reinforced polymers under different stress states are generated and compared with the test data available in the literature. For more complex three-dimensional stress states, where the test data available shows large scatter or is not available at all, a computational micro-mechanics framework is used to validate the failure criteria. It is observed that, in the case of the IM7/8552 carbon fibre-reinforced polymer, the effect of the nonlinear shear behaviour in the failure loci is negligible. In general, the failure predictions were in good agreement with previous three-dimensional failure criteria and experimental data. The computational micro-mechanics framework is shown to be a very useful tool to validate failure criteria under complex three-dimensional stress states.

1. Introduction

The effective use of polymer composite materials reinforced by unidirectional fibres in load carrying structures relies on the ability to obtain reliable predictions of the onset and propagation of the different failure mechanisms. Accordingly, the development of accurate, fully-benchmarked failure criteria that predict the onset of ply damage mechanisms in fibre-reinforced polymers (FRPs) is extremely important (Hinton et al., 2004). Indeed, this has been the subject of a great number of research studies in the literature for several years (Puck, 1969; Puck and Schneider, 1969; Tsai and Wu, 1971; Hinton and Soden, 1998; Cuntze and Freund, 2004; Dávila et al., 2005; Catalanotti et al., 2013).

Failure criteria are especially useful in providing failure envelopes from relatively simple experimental data that can be used in the analysis, design and calculation of safety factors of composite structures subjected to complex loading and boundary conditions. Global stress- (Christensen, 1997; Liu and Tsai, 1998; Sun and Tao, 1998; Hart-Smith, 1998b; Sun et al., 2002; Kuraishi et al., 2002) and strain- (Hart-Smith, 1998a) based criteria are generally "*purely empirical*" (Liu and Tsai, 1998), and their replacement by phenomenological failure criteria, with a stronger physical basis, is more and more common (Puck and Schürmann, 1998, 2002; Dávila and Camanho, 2003; Cuntze and Freund, 2004; Dávila et al., 2005; Pinho et al., 2006; Catalanotti et al., 2013). Nevertheless, it should be noted that failure criteria must be simple enough for application in engineering problems while capturing the relevant physics of the problem (Puck and Schürmann, 1998).

To meet the most common design requirements, failure criteria must be applicable at the ply, laminate and structural level. Because failure at these levels is often the consequence of an accumulation of micro-level failure events, an understanding of the micro-mechanical damage mechanisms is crucial to develop accurate and physically-based failure theories (Dávila and Camanho, 2003). Regarding the mathematical form of the failure criteria and the

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shape of the corresponding envelopes, there are some restrictions that must also be taken into account. For example, failure envelopes must be closed, in order to prevent infinite strength, and they must be convex, so that unloading from a stress state will not lead to additional failures (Liu and Tsai, 1998).

Among the phenomenological failure criteria presented in the literature, those proposed by Puck and Schürmann (1998, 2002) for transverse failure are often used by the composites' research and design communities, with several other theories appearing as extensions to these criteria (Dávila and Camanho, 2003; Dávila et al., 2005; Pinho et al., 2005, 2006; Catalanotti et al., 2013). Puck's failure criterion for matrix transverse cracking is based on a modified Mohr/Coloumb theory for brittle transversely-isotropic materials. The tractions acting on the fracture plane, which need to be determined, are used to assess failure under two fundamental regimes: transverse tension (positive normal stress) or transverse compression (negative normal stress). The orientation of the fracture plane is determined when the *plane of maximum stress effort* is found, i.e., the plane that maximises the failure index. This means that a maximisation problem needs to be solved, which is generally done by varying the orientation of the fracture plane, calculating the failure index for every angle, and recording the orientation giving the maximum value of the failure index.

In the design and analysis of thick composite laminates, general three-dimensional (3D) stress states should be accounted for, and reliable failure criteria must be formulated for these cases. Improved 3D failure criteria, based on the ideas presented by Puck and Schürmann (1998, 2002), were recently proposed by Catalanotti et al. (2013) to address more general stress states in a consistent way, providing not only the predictions for the onset of ply damage, but also additional information regarding the type of failure and the orientation of the fracture plane.

In order to obtain a simpler, but elegant description of failure of laminated composites, Cuntze and Freund (2004) presented a formulation of different failure criteria based on invariants for each individual failure mode observed in anisotropic, heterogeneous materials. This means that the formulations do not depend on coordinate-system transformations. Therefore, the search for the fracture plane is not necessary. Cuntze and Freund (2004) suggests that the results of the invariant-based failure criteria can then be used for post-determination of the angle of the fracture plane; e.g., Cuntze and Freund (2004) used Mohr/Coloumb theory, but considered only a state of plane stress, without including the shear stresses σ_{12} and σ_{13} (1 corresponds to the fibre direction, and 2–3 to the transverse directions). Cuntze and Freund (2004) claim that, besides not predicting directly the orientation of the fracture plane, the invariant approach's simplicity results in a certain loss of 'physical correctness'.

In this work, new 3D failure criteria for fibre-reinforced composite materials based on the transversely isotropic yield function used by Vogler et al. (2013) is presented (Vogler et al., 2010). The new 3D failure criteria have an invariant quadratic formulation based on structural tensors that accounts for the preferred directions of the anisotropic material. With this formulation, anisotropy is not derived using symmetry conditions based on a reference coordinate system, but by the structural tensors, which represent the anisotropy as an intrinsic material property. In other words, the anisotropic constitutive equations are represented as isotropic tensor functions, and an elegant *coordinate system-free* description of anisotropy is obtained. Note that the 3D invariant-based failure criteria proposed in this work predicts failure in a single unidirectional (UD) ply, requiring the calculation of the strains and stresses of each ply of multidirectional laminates, which is a common procedure in the most recent failure criteria for FRPs (Puck and Schürmann, 1998, 2002; Dávila and Camanho, 2003; Dávila et al., 2005; Catalanotti et al., 2013).

2. Invariant-based criterion for transverse failure

When formulating physically-based failure criteria for FRPs, it is essential to distinguish between transverse, matrix-dominated failure mechanisms, and longitudinal, fibre-dominated failure mechanisms (Puck and Schürmann, 1998). Regarding the transverse failure mechanisms, the proposed 3D invariant-based criterion is formulated directly from the yield function presented by Vogler et al. (2013). According to the invariant formulation of anisotropic constitutive equations (Boehler, 1987), transversely isotropic materials can be characterised by a preferred direction **a**. For unidirectional FRPs, this preferred direction **a** is the fibre direction (Spencer, 1987; Vogler et al., 2013). If the preferred direction coincides with the x_1-direction, which is often the case for UD fibre-reinforced laminates, the preferred direction is represented as:

$$\mathbf{a} = [1\ 0\ 0]^{\mathrm{T}} \tag{1}$$

The material response is invariant with respect to arbitrary rotations around the preferred direction **a**, to reflections at planes parallel to the fibre direction, and with respect to the reflection at that plane, whose normal is the preferred direction **a** (Vogler et al., 2013).

The structural tensor **A** of transverse isotropy, which represents the material's intrinsic characteristic direction, is an additional tensor argument to formulate the failure criterion (or yield function) as an isotropic tensor function. It is defined as the dyadic product of the preferred direction **a**, i.e. (Spencer, 1987; Vogler et al., 2013):

$$\mathbf{A} = \mathbf{a} \otimes \mathbf{a} \tag{2}$$

According to the proposed invariant formulation, the failure index f is a function of the stress tensor $\boldsymbol{\sigma}$ and of the structural tensor **A**, which are the argument tensors. The corresponding isotropic invariants form the functional basis for constructing the failure index f as a scalar isotropic tensor function (Spencer, 1987; Vogler et al., 2013).

The functional basis for transverse isotropy, formed by the argument tensors $\boldsymbol{\sigma}$ and **A** reads (Boehler, 1987; Vogler et al., 2013):

$$\mathrm{tr}\,\boldsymbol{\sigma}, \quad \mathrm{tr}\,\boldsymbol{\sigma}^2, \quad \mathrm{tr}\,\boldsymbol{\sigma}^3, \quad \mathrm{tr}(\mathbf{A}\boldsymbol{\sigma}) \quad \text{and} \quad \mathrm{tr}(\mathbf{A}\boldsymbol{\sigma}^2) \tag{3}$$

It should be noted that an arbitrary linear combination of the stress tensor $\boldsymbol{\sigma}$ can be used in Eq. (3). Furthermore, a linear combination of the basic invariants given in (3) can also be used (Vogler et al., 2013).

Vogler et al. (2013) reformulated the quadratic ($\mathrm{tr}\,\boldsymbol{\sigma}^2$ and $\mathrm{tr}(\mathbf{A}\boldsymbol{\sigma}^2)$) and the linear ($\mathrm{tr}\,\boldsymbol{\sigma}$) invariants of the functional basis (3) to identify particular stress states with the corresponding invariants. The basic invariant $\mathrm{tr}\,\boldsymbol{\sigma}^3$ is neglected; the cubic invariant $\mathrm{tr}\,\boldsymbol{\sigma}^3$ is suitable for modelling metal plasticity, which is obviously not the case; the invariant $\mathrm{tr}(\mathbf{A}\boldsymbol{\sigma})$ can be used to simulate yielding in the fibre direction, which is neglected when modelling UD fibre-reinforced laminates based on brittle resins.

Following Vogler et al. (2013), the quadratic invariants $\mathrm{tr}\,\boldsymbol{\sigma}^2$ and $\mathrm{tr}(\mathbf{A}\boldsymbol{\sigma}^2)$ of the functional basis (3) are reformulated assuming a decomposition of the stress tensor $\boldsymbol{\sigma}$ in plasticity inducing stresses $\boldsymbol{\sigma}^p$ and reaction stresses $\boldsymbol{\sigma}^r$ (Spencer, 1987):

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^p + \boldsymbol{\sigma}^r \tag{4}$$

with:

$$\boldsymbol{\sigma}^r = \frac{1}{2}(\mathrm{tr}\,\boldsymbol{\sigma} - \mathbf{a}\boldsymbol{\sigma}\mathbf{a})\mathbf{1} - \frac{1}{2}(\mathrm{tr}\,\boldsymbol{\sigma} - 3\mathbf{a}\boldsymbol{\sigma}\mathbf{a})\mathbf{A} \tag{5a}$$

$$\boldsymbol{\sigma}^p = \boldsymbol{\sigma} - \boldsymbol{\sigma}^r \tag{5b}$$