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Parametric analysis of structures from flat vaults to reciprocal grids

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ABSTRACT

We propose a parametric analysis of a class of structural systems stemming from the 17th century invention of Joseph Abeille usually called flat vault and reaching the field of reciprocal grids. Our purpose is to understand the load descent paths taking place in the various specimens of this class and their relations with basic structural principles such as that of the inverted catenary, useful to deal with vaults, or that of the lever, more appropriate for grids.

The analysis is performed on changes of the geometry and of the topology that preserve the logic of the bonding of stone blocks characterizing Abeille's invention. Shown results concern the distribution of the elastic energy, the reactions, the chirality and the stress and displacement maps.

Our findings support the idea that the structures belonging to this class of structures (and having reasonable proportions) are rather to be considered as deflected grids than as compressed vaults. Furthermore, a local bending interests the blocks, for because of the bonding, in a way that is typical of reciprocal structures.

1. Introduction

The starting point of this work is a stone structure, called a flat vault, invented by the French engineer Joseph Abeille in 1699, where the appropriate interweaving of stone blocks allows the covering of a rectangular space. In the structure designed by Abeille, the standard block has two orthogonal vertical sections in the shape of isosceles trapezia (each with one pair of sides horizontal, see Fig. 1). Some variations exist that present more complex shapes, but we will not study them here.

In the past years many researchers—especially in the field of construction history—have analyzed the mechanical behavior of these objects to give a synthetic interpretation of their nature (a bibliographical survey is presented in Brocato and Mondardini (2011); see also Fleury (2009), Nichilo (2003), Rabasa-Dìaz (1998), Sakarovitch (2006) and Uva (2003)). These interpretations are based on two main positions:

(V1) The structure behaves like a vault, where the stone elements are primarily subject to axial stress and transmit a sensible thrust on the confinement structures (catenary effect).

(V2) The structure can be seen as a kind of stacked timber grid, where elements are primarily subject to bending ('levery' effect, if one admits the neologism).

If the rationale behind the first thesis is obvious, the second is supported by a kinship between Abeille's system and a particular family of structures called nexorades or reciprocal frames (Baverel et al., 2000). In particular the structural assembly of timbers called Serlio floor (a proposal appearing in the literature since the middle-age; see Yeomans (1997)) seemingly works like Abeille's: in both examples, each element supports and is supported at the same time by other equivalent components in order to cover a space with shorter pieces than the span.

Our purpose is to understand at which rate, depending on its geometry, the structural system is represented by either model. This insight is useful in the practice of structural civil engineering, both because it helps deciding on the adequacy of the system to any particular application and because it helps assigning criteria for the design of the individual pieces, of the supports and of the abutments.

In the last decade, following a proposal by Dyskin et al. (2001) that renewed the attention on an even older theme, some authors have dealt with the subject of the topological interlocking of blocks (Dyskin et al., 2003a; Dyskin et al., 2003b; Estrin et al., 2011; Khandelwal et al., 2012), showing interest on structural systems kin to those studied here as a basis to develop new materials. In

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these papers the investigation on the mechanical properties of the system focuses predominantly on the force-deflection behavior under large displacements and until ruin. Experiments made to gather information on this behavior show evidence of the setting of a catenary effect during failure, as confirmed by comparison with some theoretical results obtained modeling the structure as an arch (Khandelwal et al., 2012).

This dominance of the catenary effect can be explained by the fact that the considered systems—unlike those examined here—are made of regular polyhedra (especially tetrahedra), i.e. elements that have no dimension prevailing on the others. Hence our investigation might contribute extending the picture put forward in the quoted papers.

It must be noticed that Khandelwal et al. (2012) seems to deduce from the literature on Abeille's bonds that these flat vaults do not withstand loads of inverted direction and work only under a stabilizing self-weight. This is actually not the case: Abeille's flat vaults can withstand orthogonal and transversal loads in all directions and do not need be stabilized by their weight.

An investigation on a similar system is presented in Brocato and Mondardini (2012), where spherical domes are considered and a method to define an optimal structure is proposed. Here the study of flat vaults is developed through the generation of three parametric families of topologically equal structures, plus one family of topological variants.

When considering the two previously listed viewpoints (V1 and V2), clearly, the first can be expected to be more consistent of the second if the elements composing the structure are bulky, less if they are slender. A slenderness parameter, given by the ratio of the horizontal dimensions of the blocks, can be introduced to help

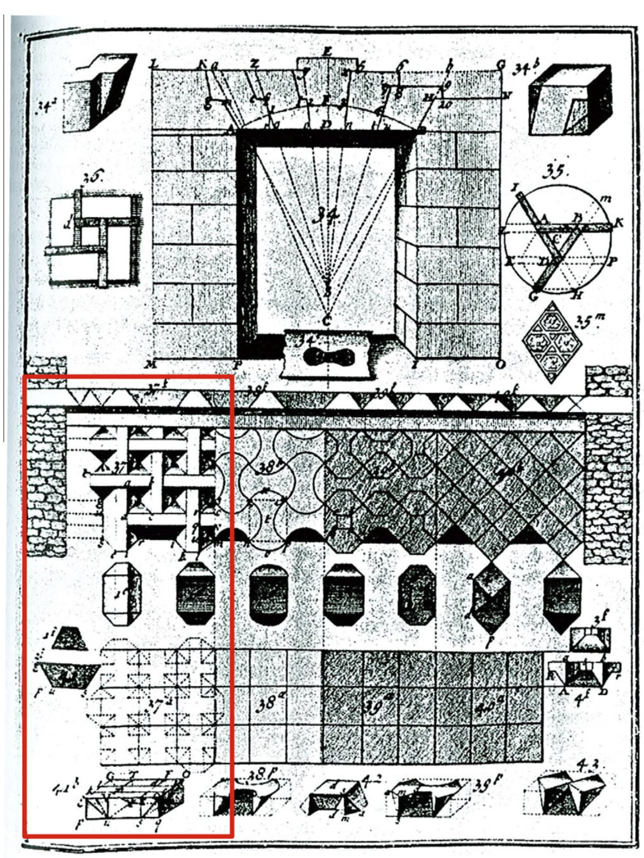

Fig. 1. Examples of flat structures presented by Frézier, 1980. Abeille's proposal appears in the red box. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

gauging this issue. Similarly, considering that the catenary effect is likely to be more efficient when the thickness of the structure grows with respect to the span, less in the opposite condition, a thickness parameter is introduced, given by the ratio between the thickness of the vault and its overall length. Also the inclination of the contact planes between blocks can be assumed to contribute, when they are closer to vertical planes, more to the behavior of a catenary system than to that of a 'levery' one and vice versa in the opposite condition. This wedge effect can be measured by what we name the 'splice angle', i.e., the inclination from the vertical of the legs of the trapezia describing the cross sections of blocks.

The chiral geometry of the considered structures has a consequence on the nature of their thrust, which, under a symmetric load, is necessarily expressed by a chiral set of force vectors. Nevertheless this effect should diminish when the number of pieces composing the vault grows, as the characteristic length of the chirality becomes much smaller than the span. A parameter that can be introduced to represent the importance of chirality is the number of stone rows the structure is made of or, equivalently, the ratio of the distance between two such rows on the span.

Hence, we have taken into account the four quoted parameters to generate several structures bearing equivalent loads with equivalent boundary conditions and study their variations.

To evaluate the difference between members of these families, the definition of the shear, bending and membrane energies within the structure is adapted to the case and their quotas in terms of total elastic energy computed for a given structure. Then these quotas are compared for flat vaults having different slendernesses and splice angles, showing that the influence of the latter on the behavior of the vault is but slightly important. The same quotas are compared for vaults with different number of rows and for vaults with different thickness parameter. Maps of the bending part of the elastic energy are also proposed to show the difference between the structure considered here and a grid of continuous interlacing beams.

In the next section the parametric families of considered structures, the model used for numerical simulations, and the different measures used to indicate the mechanical performances of the vault are presented. The numerical results are given, in Section 3, with a discussion on these performances, considering four different kind of analyses, which focus respectively on the distribution of the elastic energy, the boundary reactions, the geometric chirality and its mechanical effect, and a class of topological variations of the system.

2. Assumptions

2.1. Parametric geometry

The geometry of blocks is presented in Fig. 2. Any cross section of the block orthogonal to the $2a$ and $2d$ long edges has the shape of an isosceles trapezium; the same happens for the cross sections orthogonal to the $2c$ and $2b$ edges. The splice angle φ is the angle between the vertical and the legs of these trapezia, and it is the same for all cross sections in both x- and y-direction (see Fig. 2). The height of these cross sections, or structural thickness of the vault, is denoted by h. These six measures are related by the two conditions

$$c = b + h\tan\varphi, \quad d = a - h\tan\varphi, \tag{1}$$

so that two of them, namely c and d, will in this paper be always considered as dependent variables defined by (1).

The overall length L and the span S of the vault indicated in Fig. 3 are related to the even integer number $2N$ representing the number of rows spanning in the orthogonal direction by the equations: