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Decoupling transformation for piezoelectric–piezomagnetic fibrous composites with imperfect interfaces

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ABSTRACT

We study the effective magnetoelectricity of piezoelectric–piezomagnetic fibrous composites with imperfect interfaces subjected to anti-plane shear deformation coupled to in-plane electromagnetic fields. Two kinds of imperfect interfaces, mechanically stiff and highly electromagnetic conducting interfaces, and mechanically compliant and weakly electromagnetic conducting interfaces, are considered for the phenomenon of contact resistance between the constituents. The decoupling transformation approach previously used in composite media with perfect interfaces is extended to the current configuration. The predicted macroscopic behaviors are in good agreement with the known solutions. It is observed that moduli of the composite with imperfect contacts do not fulfill the compatibility conditions $\mathbf{L}^*\mathbf{L}_a^{-1}\mathbf{L}_b - \mathbf{L}_b\mathbf{L}_a^{-1}\mathbf{L}^* = \mathbf{0}$, which was supposed to be microstructure independent connections for the two-phase heterogeneous media. Finally, we use the two-level recursive scheme to show the equivalence between the imperfect interfaces and an infinity thin homogeneous interface layer.

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1. Introduction

The subject of the present paper is exploiting the decoupling transformation to study the effective moduli of multiferroic fibrous composites with imperfect interfaces. We consider a piezoelectric-piezomagnetic two-phase composite under an anti-plane shear deformation coupled to in-plane electromagnetic fields. The constituents are transversely isotropic and are with circular cylindrical microgeometry. Two kinds of imperfect interfaces are considered: (i) mechanically stiff and highly electromagnetic conducting interfaces, and (ii) mechanically compliant and weakly electromagnetic conducting interfaces.

The decoupling transformation was first proposed by Straley (1981) who showed that the thermoelectric problem in a two-phase composite can be transformed into a simpler uncoupled conduction problem of an inhomogeneous medium with the same microgeometry. Milgrom and Shtrikman (1989a) and Milgrom and Shtrikman (1989b) later showed that this transformation is in fact a special case of the field decoupling transformation that can be applied to transport problems in two-phase composites with multi-coupled fields. Further, they found that one of the consequence of this mapping is that the overall response of a composite made of two constituents must obey a number of compatibility relations, which are independent of the microstructure of the media and the volume fractions of the phases. The finding is based on the fact that there exists a congruent transformation which simultaneously diagonalizes the two-phase material property matrices into an eigenbasis. Benveniste (1994a), Benveniste (1994b) and Benveniste (1997) extended this idea to the piezoelectric composite or polycrystalline aggregates subjected to the anti-plane shear deformation with in-plane electric intensity. Following this line, Benveniste (1995) applied it to the piezoelectric-piezomagnetic composite. Related subjects of this kind also include Chen (1993), Chen (1997) and Nan (1994).

Almost all of the existing works using decoupling transformation adopt the assumption of ideal coupling between the constituents. In the context of magnetoelectricity, this assumption implies that the potential fields (displacement, electric potential, magnetic potential) and the normal component of the fluxes (stress, electric displacement, magnetic flux) are assumed to be continuous at phase interfaces. Imperfect interfaces may, however, be present in many circumstances such as roughness, debonding, sliding or cracking at the common boundary. Non-ideal interfaces may exhibit a discontinuity in the normal component of the flux but yet maintaining the continuity of the potential fields. It may arise due to the presence of a thin interphase layer of stiff elasticity and highly electromagnetic conductivity. This kind of interface assumes the discontinuity of the normal component of the flux is proportional to the surface Laplacian of the potential field at the contact boundary (Pan et al., 2009; Kuo, 2013). Another kind of non-ideal interfaces is that in which the normal component of

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the flux is continuous, but the potential field suffers a jump across it. Such behavior arises, for example, in the presence of a thin interphase of compliant elasticity and weakly electromagnetic conductivity. In this study, this kind of contact resistance is modeled by assuming that the discontinuity in the potential field is proportional to the normal component of the flux by means of an interface parameter (Wang and Pan, 2007; Kuo, 2013).

In 2013, Kuo studied the similar problems with the same types of imperfect interfaces. In that work, Kuo extended the classic work of Rayleigh (1892) in a periodic conductive perfect composite to the coupled piezoelectric-piezomagnetic composite with imperfect interfaces. This approach is an exact method that provides the detailed local field distributions of the heterogeneous media. The result was then applied to estimate the effective behaviors of the composite. In this paper, however, we propose a different approach focusing on the extension of the decoupling transformation to the proposed problem. In contrast to the exact method proposed by Kuo (2013), here the micromechanical model is an approximate approach based on a single inclusion, and focus on the overall properties of the inhomogeneous media.

The plan of this article is organized as follows. We consider a composite medium made of piezoelectric and piezomagnetic phases arranged in a microstructure consisting of parallel cylinders in a matrix in Section 2. The composite is subjected to anti-plane shear mode of deformation coupled to in-plane electromagnetic fields. The interface contacts of the constituents are imperfect: either mechanically stiff and highly electromagnetic conducting interfaces, or mechanically compliant and weakly electromagnetic conducting interfaces. In Section 3, the decoupling transformation is proposed which maps the field equations into an equivalent class of conductive heterogeneous media. We obtain effective moduli in Section 4. Numerical results are shown and compared with those obtained by other methods in Section 5. Further, we find that moduli of the composite with imperfect contacts do not fulfill the compatibility conditions $\mathbf{L}^*\mathbf{L}_a^{-1}\mathbf{L}_b - \mathbf{L}_b\mathbf{L}_a^{-1}\mathbf{L}^* = \mathbf{0}$, which was supposed to be microstructure independent connections for the two-phase composite. We use the two-level recursive scheme with the Mori–Tanaka method to show that the equivalence between the two-phase composite with imperfect interfaces and the three-phase heterogeneous media.

2. General setting

We consider a two-phase magnetoelectroelastic media subjected to anti-plane shear strains $\gamma_{zx}^0,\ \gamma_{zy}^0$, in-plane electric fields $E_x^0,\ E_y^0$ and magnetic fields $H_x^0,\ H_y^0$ at infinity. The composite therefore is in a state of generalized anti-plane shear mode of deformation and can be expressed by (Benveniste, 1995)

$$\begin{aligned} &u_x = u_y = 0, \quad u_z = w(x,y), \\ &\varphi = \varphi(x,y), \\ &\psi = \psi(x,y). \end{aligned} \tag{1}$$

Here $u_x,\ u_y,\ u_z$ are the mechanical displacements along the x-, y-, and z-axes, and φ and ψ are the electric and magnetic potentials, respectively. The constitutive laws of the constituents for the non-vanishing fields can be recast in the compact form

$$\mathbf{\Sigma}_j^{(k)} = \mathbf{L}^{(k)}\mathbf{Z}_j^{(k)}, \quad j = x, y, \tag{2}$$

where

$$\mathbf{\Sigma}_j^{(k)} = \begin{pmatrix} \sigma_{zj} \\ D_j \\ B_j \end{pmatrix}^{(k)}, \quad \mathbf{L}_k = \begin{pmatrix} C_{44} & e_{15} & q_{15} \\ e_{15} & -\kappa_{11} & -\lambda_{11} \\ q_{15} & -\lambda_{11} & -\mu_{11} \end{pmatrix}^{(k)}, \quad \mathbf{Z}_j^{(k)} = \begin{pmatrix} \gamma_{zj} \\ -E_j \\ -H_j \end{pmatrix}^{(k)}. \tag{3}$$

The superscript k refers to the matrix (m) or inclusion (i) phase. In Eq. (3), $\sigma_{zj},\ D_j,\ B_j$ are the shear stress, electric displacement and magnetic flux; $\gamma_{zj},\ E_j,\ H_j$ are the shear strain, electric field and magnetic field vectors. The material constants $C_{44},\ \kappa_{11},\ \mu_{11},\ \lambda_{11}$ are the elastic modulus, dielectric permittivity, magnetic permeability, and the magnetoelectric coupling coefficient, while e_{15} and q_{15} are the piezoelectric and piezomagnetic coefficients.

The shear strains γ_{zx} and γ_{zy}, in-plane electric fields E_x and E_y, and in-plane magnetic fields H_x and H_y can be derived from the gradient of vertical elastic displacement w, electric potential φ, and magnetic potential ψ as follows:

$$\gamma_{zj} = w_{,j}, \quad -E_j = \varphi_{,j}, \quad -H_j = \psi_{,j}, \tag{4}$$

where the subscript j following a comma denotes the derivative with respect to x or y. In the absence of body force, electric charge density and electric current density, the equilibrium equations are

$$\sigma_{zj,j} = 0, \quad D_{j,j} = 0, \quad B_{j,j} = 0. \tag{5}$$

Substitution of Eqs. (2) and (4) into Eq. (5) yields

$$\begin{aligned} &C_{44}\nabla^2 w + e_{15}\nabla^2\varphi + q_{15}\nabla^2\psi = 0, \\ &e_{15}\nabla^2 w - \kappa_{11}\nabla^2\varphi - \lambda_{11}\nabla^2\psi = 0, \\ &q_{15}\nabla^2 w - \lambda_{11}\nabla^2\varphi - \mu_{11}\nabla^2\psi = 0, \end{aligned} \tag{6}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ represents the two-dimensional Laplace operator for the variable x and y.

In addition to these differential equations, we have to use interface conditions. Two kinds of imperfect contacts of the constituents are studied. First, consider that the interface is a mechanically stiff and highly electromagnetic conducting interface, which is a generalization of the interface stress model (GISM). It assumes that the potential $\mathbf{\Phi}$ is continuous across the interface ∂V, while there is a jump in the normal component of the current $\mathbf{\Sigma}_j^{(m)}n_j$ (Miloh and Benveniste, 1999; Pan et al., 2009). Specifically, one has

$$\mathbf{\Sigma}_j^{(i)}n_j\Big|_{\partial V} - \mathbf{\Sigma}_j^{(m)}n_j\Big|_{\partial V} = \boldsymbol{\alpha}\nabla_s^2\mathbf{\Phi}^{(i)}\Big|_{\partial V}, \quad \mathbf{\Phi}^{(m)}\big|_{\partial V} = \mathbf{\Phi}^{(i)}\big|_{\partial V} \tag{7}$$

with

$$\boldsymbol{\alpha} = \lim_{\substack{t\to 0 \\ \mathbf{L}_c^{-1}\to\mathbf{0}}}(t\mathbf{L}_c) = \begin{pmatrix} \alpha^w & 0 & 0 \\ 0 & \alpha^\varphi & 0 \\ 0 & 0 & \alpha^\psi \end{pmatrix}. \tag{8}$$

Here $\mathbf{\Phi} = (w, \varphi, \psi)^T, T$ denotes the transpose of the vector, $\nabla_s^2 = \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$ is the surface Laplace operator, $\mathbf{n}$ is the unit outward normal to the interface ∂V, and the repeated index j denotes the summation over the components x and y. The imperfect interface is modeled by a concentric elastic coating of thickness t and material property $\mathbf{L}_c = diag(C_{44}^{(c)},\ -\kappa_{11}^{(c)},\ -\mu_{11}^{(c)})$ (Torquato and Rintoul, 1995; Hashin, 2001; Miloh and Benveniste, 1999). When $\boldsymbol{\alpha} = \mathbf{0}$, it corresponds to a perfect interface, while $\boldsymbol{\alpha}^{-1} = \mathbf{0}$ corresponds to an isoexpansion and equipotential interface.

Another kind of non-ideal interfaces is a mechanically compliant and weakly electromagnetic conducting interface, which is a generalization of the linear spring model (GLSM). In this case, the potential $\mathbf{\Phi}$ has a jump on the interface boundary ∂V, while the normal component $\mathbf{\Sigma}_j n_j$ of the current is continuous across the interface (Miloh and Benveniste, 1999; Wang and Pan, 2007),

$$\mathbf{\Sigma}_j^{(m)}n_j\Big|_{\partial V} = \mathbf{\Sigma}_j^{(i)}n_j\Big|_{\partial V}, \quad \mathbf{\Phi}^{(m)}\big|_{\partial V} - \mathbf{\Phi}^{(i)}\big|_{\partial V} = \boldsymbol{\beta}\mathbf{\Sigma}_j^{(i)}n_j\Big|_{\partial V} \tag{9}$$

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