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Hydromechanical modeling of an initial boundary value problem: Studies of non-uniqueness with a second gradient continuum

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ABSTRACT

A non-uniqueness study for a hydromechanical boundary value problem is performed. A fully saturated porous medium is considered using two different elasto-plastic constitutive equations to describe the mechanical behavior of the skeleton. Both models are based on the Drucker–Prager yield criterion with a hyperbolic hardening rule for the cohesion and friction angle as a function of an equivalent plastic strain. The two constitutive equations taken into account (Plasol and Aniso-Plasol) differ only for the elastic part of the model: isotropic elasticity or cross anisotropic elasticity respectively. A real hydromechanical experiment which consists in a hollow cylinder test on a Boom Clay sample is modeled in two phases. For the first phase (a hydromechanical unloading) non-uniqueness studies are carried out using both constitutive equations. In the second phase, boundary conditions are kept constant to dissipate the excess water pressure. It is shown in the first phase that the time step discretization of the numerical problem has an effect on the initialization of the Newton–Raphson algorithm on a given time step. Different solutions for the same initial boundary value problem can consequently be found. A convergence study is also presented giving an insight into the behavior of the computation during the iterations.

1. Introduction

In the simulation of initial boundary value problems using constitutive equations for geomaterial behavior, it is well known that some difficulties can arise, particularly if degradation of the materials occurs. These problems have been studied for the case of single phase materials and some theoretical results have been established. Within the small strain assumption and for a rate (or incremental) problem, the uniqueness can be proven using the so-called Hill exclusion functional (Hill, 1978). In particular, considering some additional hypothesis, the uniqueness of a rate problem can be ensured when the second order work is positive everywhere and for any strain field (see Hill, 1958, 1959). This result which gives a link between the monotonicity condition involved in the Hill exclusion functional condition and the positiveness of the second order work is far from general. It holds for instance for classical isotropic hardening elastoplastic and hypoplastic models (see Chambon and Caillerie, 1999).

On the other hand, it has been clearly proven that some general features (such as the non-associativeness) of constitutive equations induce the possibility of negativeness of the second order work (see Mroz, 1963; Raniecki and Bruhns, 1981; Bigoni and Hueckel, 1991). It is for this reason that computations of geomaterial structures cause more difficulties than computations with simple materials such as metals. An extensive review of the literature concerning non-uniqueness can be found in Petryk (1997) and Bigoni (2000) (see also Bigoni, 2012). Nevertheless, it is worth mentioning studies concerning localization (Rice, 1976) and controllability (Nova, 1994; Chambon et al., 2004, Chambon, 2005) which can be seen as very particular cases of uniqueness studies in rate problems (in both cases at least one of the non-unique solutions is homogeneous, as expected).

Unfortunately, the theoretical results cannot be applied for all kinds of constitutive equations, including the implicit ones obtained by numerical homogenization in the so-called multiscale computations (see Kouznetsova et al. (2001) and Frey et al. (2012) for coupled problems). To have a good insight into the non-uniqueness problem of an initial boundary value problem, one can efficiently study the loss of uniqueness using numerical tools.

If an implicit method is applied to solve the problem for a loading step (or the rate problem if this loading step tends towards zero), a full Newton–Raphson method (whose solution can depend

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on the initial guess) is usually used since this problem is still non-linear due to the possibility for many points of the computed body to load or unload. More generally, from a numerical standpoint, in case of non-uniqueness, the solution can be influenced by different numerical input such as the spatial discretization of the problem, the size of the time step, the mesh spacing, some tolerance values, and so on (Chambon et al., 2001b).

It is well known that the problems regarding loss of uniqueness of the solution of initial boundary value problems can be related to the triggering of localized strains. The classical non-viscous constitutive equations cannot describe properly the development of the localized deformations since this implies rupture without energy consumption and a strong mesh dependency of the numerical results (see Bazant et al., 1984). To avoid this non-physical behavior, an internal length has to be incorporated in the modeling. In the past, several approaches were proposed to formulate this kind of enhanced models. Let us, for instance, cite non-local models (Pijaudier-Cabot and Bazant, 1987), strain gradient plasticity theories (Aifantis, 1984, 1987) and higher order continua theories (see Forest and Sievert (2006) for a complete review).

Unlike the above theories, it is worth mentioning the work of Gajo et al. (2004) in which, following the main idea of Hutchinson and Tvergaard (1981) and Petryk and Thermann (2002), the post-localization analysis is performed enforcing directly the thickness and the inclination of the shear bands.

In the framework of microstructure continuum theory formulated by Germain (1973b), second gradient models (Chambon et al., 2001a) can be considered to solve mesh-dependency of an initial boundary value problem when localized strains appear. Despite the fact that an internal length is incorporated in the modeling, this is not sufficient to avoid the occurrence of multiple solutions for the same problem (see Chambon et al., 1998). Generalizing to second gradient models the numerical method described previously for classical model, the non-uniqueness has been clearly demonstrated in many papers (Chambon and Moullet (2004) or Bésuelle et al. (2006) for instance). A principal conclusion of these numerical investigations is that as soon as the monotonicity condition is lost for the classical part of the model, a non-uniqueness study has to be performed.

Since geomaterials are multi-phase, it is necessary to take into account not only the solid skeleton (with its balance of momentum and constitutive equation) but also the fluid phase (and its corresponding equations) as well as the coupling between them. The resulting initial boundary value problem could perhaps restore uniqueness due to the properties of the Darcy law involved in the coupled problem, which, some argue, intrinsically introduce an internal length. One of the objectives of this paper is to show that this way of thinking is erroneous. For the hydro-mechanical coupled problem studied in this paper, numerical experiments are performed varying certain numerical inputs (the size of the time step) in order to obtain several solutions, in the same way as has been done for a single phase material (see Sieffert et al., 2009). A local hydromechanical second gradient model (see Collin et al., 2006) is used in this work to guaranty the objectivity of the solutions. It is important to highlight that, in the work of Collin et al. (2006) the second gradient effects are related only to the solid skeleton. A more general formulation is derived by Sciarra et al. (2007) in which the second gradient effects are associated also to the fluid phase.

There are few theoretical works dealing with the possible non-uniqueness of solutions for a coupled problem. It is worth mentioning the pioneering works of Rice (1975), Loret and Prevost (1991), Schrefler et al. (1995), Runesson et al. (1996), Vardoulakis (1996a,b) and Armero (1999). Some studies have been performed concerning the instability of some solutions starting from a homogeneous state by Benallal and Comi (2002, 2003). Similarly, some localization analysis for coupled problems have been investigated by Zhang and Schrefler (2001) and Schrefler et al. (2006).

Some other studies regarding the strain localization in saturated porous media are carried out by Callari and Armero (2002) and Andrade and Borja (2007). In the former work, the shear bands are modeled using the enhanced finite element method. In this method, strong discontinuities are introduced in the unknown fields at the enhanced element level. In the latter work, generalizing the formulations of Li et al. (2004) and Armero (1999), the behavior of shear bands for loose and dense sands under globally undrained conditions is simulated and the influence of a nonhomogeneous initial porosity field on the localized solution is analyzed.

More recently, to study the hydromechanical behavior of deep tunnel, some localized solutions are investigated by Plassart et al. (2013) in which, the so-called second gradient dilation (Fernandes et al., 2008) is considered to regularize the numerical problem.

Let us finally mention the works concerning controllability in a coupled context done by Imposimato and Nova (1998), Buscarnera and Nova (2011) and Mihalache and Buscarnera (2014) (see also Buscarnera and Prisco, 2012). All these theoretical studies are restricted to initially homogeneous bodies and used a linearization of the constitutive equations discarding possible unloading behavior. The use of a so-called comparison solid is clearly not mathematically justified, and moreover it is not clear that instability and non-uniqueness of the solutions are linked for problem using constitutive equations involving non associativeness like the ones used for geomaterials. However, such studies are useful and can be seen as heuristic ones.

In this study, only a numerical problem is examined and there is no proof that the behavior of the underlying mathematical problems is the same as the one of the numerical computations. We are able to investigate cases for which the initial state is not homogeneous and also to take into account unloading as well as loading. It is another heuristic manner to tackle this important problem of the validity of the continuum modeling of two-phase materials.

The work is organized as follows: in Section 2, the assumptions made and the balance equations are recalled, then the equations solved for a time step are established. In the subsequent section, the constitutive equations and the parameters used are given. Two sets of computations have been studied, one using an isotropic constitutive equation and another using an anisotropic constitutive equation. Section 4 describes the initial boundary value problem. It has to be emphasized that since we are solving a fully coupled problem, the results depend on time and on the relationship between permeability and loading velocity. Realistic parameters have therefore been chosen to simulate a real experiment, since unrealistic parameters could cause adverse interaction due to the coupling in the model. The methods used to obtain alternative solutions to the initial problem are described in Section 5. The non-uniqueness study using the isotropic constitutive equation is detailed in Section 6. Similarly, the anisotropic case can be seen in Section 7. Finally, Section 8 is devoted to results regarding the pore pressure.

2. Local hydromechanical second-gradient model

Let us first recall the main features of local second gradient models. An enriched kinematical description of the continuum is used as proposed by Germain (1973a,b). In addition to the displacement field u_i, a second order tensor (the so-called micro kinematic gradient) ν_{ij}, is introduced. Starting from this model, it is possible to restrict the kinematics by enforcing the micro gradient ν_{ij} to be equal to the macro gradient $\partial u_i/\partial x_j$, thus obtaining a local second gradient continuum medium (Mindlin, 1964;