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# Three dimensional homogenization of masonry structures with building blocks of finite strength: A closed form strength domain

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The present paper provides a straightforward methodology for the estimation in closed form of the overall strength domain of an in-plane loaded masonry wall by accounting for the failure of its bricks. The determination of the overall strength domain was based on a rigorous definition of the microstructure in, three dimensions on convex analysis and on the kinematical approach in the frame of limit analysis theory. No plane stress or plane strain assumption is a priori made. The formulation allowed distinguishing the yield surfaces that account for the failure of the joints and the yield surfaces that account for the failure of the building blocks. The validity and the efficiency of the derived analytical strength domain were investigated by means of numerical homogenization and experimental evidence. The proposed strength domain can be used in limit analysis approaches, in finite element simulations and for calibrating existing phenomenological models for masonry structures based on the micromechanical properties and the geometry of the bricks and the mortar.



## 1. Introduction

The failure of masonry structures can be studied either by continuum or discrete type models (cf. macro-modeling and micro-modeling Lourenço, 1996). The latter consider the masonry as an assemblage of blocks (bricks) with explicitly defined geometry and joints (interfaces), while the former consider the masonry as a continuum medium. Continuum models are based on either simplified analytical approaches or on homogenization techniques. Each approach has advantages and disadvantages that are related to the required computational effort and the degree of accuracy of the obtained results. Due to the heterogeneous nature of masonry structures, discrete type approaches seem to be the physical starting point for the modeling of the mechanical behavior of such kind of structures. Nevertheless, because of the difficulty in determining the exact mechanical parameters at the microlevel and the considerable computational cost of discrete type approaches, continuum approaches continue to attract the interest of many researchers. In spite of the several limitations of continuum mechanics for modeling such kind of heterogeneous systems (at least for classical Cauchy continua (Zucchini and Lourenço, 2007)) the main reason for using continuum models is that they offer a certain degree of abstraction and allow to up-scale the micromechanical characteristics to the macroscale, i.e. to the scale of the structure.

A considerable number of continuum models for masonry already exist in the literature. Among others we refer to the works of Heyman (1966), Page (1978), Livesley (1978), Alpa and Monetto (1994), Pande et al. (1989), Lotfi and Shing (1991), Pietruszczak and Niu (1992), Cecchi and Sab (2002), Zucchini and Lourenço (2002, 2007) and Milani et al. (2006,) in the frame of classical continuum theory and to Sulem and Muhlhaus (1997), Masiani and Trovalusci (1996), Stefanou et al. (2008, 2010), Salerno and de Felice (2009), Addessi et al. (2010), Pau and Trovalusci (2012) and Trovalusci and Pau (2013) for continuum models using higher order continuum theories. For a comprehensive review of various continuum models we refer to the article of Lourenço et al. (2007). As a general remark one could state that most of the available continuum models describe the elastic linear behavior of brickwork by proposing even closed form expressions for the elastic moduli. On the contrary, the inelastic behavior of masonry is studied in fewer works through non-linear homogenization approaches that in most of the cases are based on extensive numerical simulations.

Homogenization theory (Bakhvalov and Panasenko, 1989; Bensoussan et al., 1978) has been applied in order to derive the

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effective linear elastic constitutive parameters of an equivalent Cauchy continuum based on the microstructure of the masonry. Based on a kinematic limit analysis homogenization approach and under plane stress conditions, de Buhan and de Felice (1997) have derived in closed-form the strength domain of an in-plane loaded periodic brickwork consisting of infinitely resistant (elastic) bricks connected with Coulomb interfaces. The derived yield criteria consist an upper bound of the strength domain. Considering a polynomial distribution of the stresses and a two dimensional stress field (imposed plane stress conditions), Milani et al. (2006), proposed a homogenization scheme in order to determine a lower bound of the strength domain of masonry. The aforementioned homogenization approach allowed to consider different yield criteria for bricks and mortar. Massart et al. (2005, 2007) and Zucchini and Lourenço (2002, 2004, 2007) considered additionally the brittle behavior of bricks and mortar in the frame of damage mechanics theory. Nevertheless, the strength domain in the abovementioned approaches does not have an analytical, closed form expression.

The present paper focuses on providing a straightforward methodology for the analytical, closed-form estimation of the overall strength domain of an in-plane loaded masonry wall made of bricks of finite strength connected with frictional interfaces. The determination of the overall in-plane strength domain was based on a kinematic limit analysis approach in three dimensions (3D). It has to be emphasized that the common plane stress or the plane strain or the generalized plane strain assumptions were avoided (these terms are used as defined in Saada (1974)). According to Anthoine (1997), the aforementioned states of plane deformation might have little influence on the macroscopic elastic behavior of masonry (Addessi and Sacco, 2014; Mistler et al., 2007), but may significantly affect its non-linear response (at least for materials described in the damage mechanics framework, which was used in Anthoine, 1997). Therefore, the three dimensional kinematic approach followed here permits the generalization and extension of the results of de Buhan and de Felice (1997) by taking into account the out-of-plane deformations of the masonry due to in-plane loading and by considering a finite strength for the blocks. Depending on the constitutive behavior of the masonry units and of the joints, an analytical closed form expression for the masonry strength domain is determined.

The kinematic approach leads, in principle, to an upper bound of the exact strength domain of the system (cf. Salençon, 1990a,b). Therefore, the accuracy of the abovementioned analytically derived strength domain was investigated through numerical homogenization of the 3D unit cell and it was compared to the experimental results of Page (1981,1983). The effect of the thickness of the joints was explored and its influence was found to be quite limited for thin joints.

The paper has the following structure. In Section 2 the overall in-plane strength domain of a running bond masonry wall is determined based on a kinematic limit analysis approach and using a three-dimensional stress and kinematic field. In this section the formulation is general, no plane stress assumption is made and not any particular material is chosen for the interfaces and the building blocks. The masonry wall is treated as a thin plate with a periodic microstructure of finite thickness. In Section 3, the derived strength domain is compared to the strength domain found by numerical homogenization. The interfaces and the blocks are considered to obey to a Drucker–Prager criterion in order to avoid possible numerical problems. Three yield surfaces that account for the failure of the joints, and one yield surface that accounts for the failure of the units are expressed in closed form. Their intersection in the stress space forms the in-plane strength domain, which is compared with the strength domain derived numerically. It is shown that the numerical and the analytical results coincide in the majority of biaxial load configurations tested. Nevertheless, under some biaxial load conditions and for thick joints the resistance of the masonry is somehow overestimated. Finally, in Section 4 the analytically derived strength domain is compared to the experimental results of Page (1981,1983) by adopting a Coulomb criterion both for the interfaces and the blocks. The comparison is quite satisfactory.

The derived analytical strength domain can be used in limit analyses in order to assess the ultimate failure load, in finite element simulations (e.g. De Felice et al., 2009) and due to its simple closed-form expression can be used for the calibration of existing phenomenological models (e.g. Ottosen, 1977; Syrmakezis and Asteris, 2001).

## 2. Three dimensional homogenization of masonry walls

Homogenization theory is applied in order to determine the overall in-plane strength domain of a running bond masonry wall. A kinematic limit analysis approach is followed using a three-dimensional stress and kinematic field. It is worth emphasizing that unlike similar existing homogenization approaches for masonry (e.g. de Buhan and de Felice, 1997; Milani et al., 2006), no plane stress conditions are a priori assumed and the problem is treated in three dimensions. The reason is that the stress state in the mortar cannot be described precisely either by plane stress or plane strain conditions. In particular, one can imagine that when the joints are very thin the mortar is in plane strain conditions as the masonry units constrain its deformation. On the contrary, when the joints are very thick, the influence of the units on the deformation of the mortar is small and one can consider that the mortar deforms rather under plane stress conditions. Following the definitions of Saada (1974), in the absence of lateral loadings, a masonry wall is in a generalized plane stress state, i.e. the stress is zero at its lateral sides, but not in every point in its thickness (cf. plane stress conditions). The influence of plane stress or of generalized plane strain conditions is well-known (Anthoine, 1997; Mistler et al., 2007) and in the non-elastic regime, different states of plane deformation can have important impact. Generalized plane strain and simplified 3D approximations give better results as far it concerns the resistance of masonry (Addessi and Sacco, 2014; Anthoine, 1995). The plane stress assumption is inadequate for thick masonry walls (Anthoine, 1997). To overcome these issues a three dimensional kinematic and stress field is taken into account and the masonry wall is considered as a plate of finite thickness.

Let the heterogeneous plate occupy the domain $\Omega = \omega \times \left]-\frac{t}{2}, \frac{t}{2}\right[$ where $\omega \subset \mathbb{R}^2$ is the middle surface (middle plane) and $t$ the thickness of the plate. The plate consists of an elementary (unit) cell that it is periodically repeated in directions 1 and 2 (see Figs. 1 and 2) and its size is small in comparison to the size of the total structure. The elementary unit cell is denoted by the domain $Y = A \times \left]-\frac{t}{2}, \frac{t}{2}\right[$, where $A \subset \mathbb{R}^2$. The boundary $\partial Y$ of $Y$ is decomposed into three parts, $\partial Y = \partial Y_l \cup \partial Y_3^+ \cup \partial Y_3^-$, with $\partial Y_3^{\pm} = \left\{\pm\frac{t}{2}\right\}$.

We assume that the strength of the material at every point $\mathbf{y} \in Y$ is defined by a convex closed domain $G(\mathbf{y})$, such that $\boldsymbol{\sigma} \in G$, with $\boldsymbol{\sigma} = (\sigma_{ij})$ the stress tensor and $i,j = 1,2,3$. No plane stress assumption is made and therefore $\sigma_{13}, \sigma_{23}$ and $\sigma_{33}$ are not zero. Such a domain is uniquely defined by a positive homogeneous function of degree one, which is called support function and it is defined as:

$$\pi(\mathbf{d}) = sup\{\boldsymbol{\sigma} : \mathbf{d}, \boldsymbol{\sigma} \in G\} \Longleftrightarrow G = \{\boldsymbol{\sigma} | \boldsymbol{\sigma} : \mathbf{d} \leqslant \pi(\mathbf{d}), \forall \mathbf{d}\} \quad (1)$$

where $\mathbf{d} = (d_{ij})$ denotes a strain rate tensor and ‘:’ denotes the double contraction.