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A thermodynamically-consistent microplane model for shape memory alloys

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ABSTRACT

In microplane theory, it is assumed that a macroscopic stress tensor is projected to the microplane stresses. It is also assumed that 1D constitutive laws are defined for associated stress and strain components on all microplanes passing through a material point. The macroscopic strain tensor is obtained by strain integration on microplanes of all orientations at a point by using a homogenization process. Traditionally, microplane formulation has been based on the Volumetric–Deviatoric–Tangential split and macroscopic strain tensor was derived using the principle of complementary virtual work. It has been shown that this formulation could violate the second law of thermodynamics in some loading conditions. The present paper focuses on modeling of shape memory alloys using microplane formulation in a thermodynamically-consistent framework. To this end, a free energy potential is defined at the microplane level. Integrating this potential over all orientations provides the macroscopic free energy. Based on this free energy, a new formulation based on Volumetric–Deviatoric split is proposed. This formulation in a thermodynamic-consistent framework captures the behavior of shape memory alloys. Using experimental results for various loading conditions, the validity of the model has been verified.

1. Introduction

Shape memory alloys (SMAs) are finding increasing number of engineering applications due to their unique properties. In general, there are two main approaches for modeling the complex behavior of SMAs, micromechanical and macromechanical. In general, a micro-scale viewpoint can result in a more accurate understanding of the material behavior. In micromechanical models, the micro-scale response of SMAs is investigated by considering the grain level of the two phases and the crystallographic texture of the material. These models use thermodynamics laws and micromechanics methods to describe the transformation and macro-scale equations (Gao et al., 2000; Auricchio et al., 2003; Thamburaja, 2005; Guthikonda et al., 2007; Sadjadpour and Bhattacharya, 2007; Peng et al., 2008; Yu et al., 2013). In macromechanical models, on the other hand, macro-scale behavior is captured by considering macroscopic energy functions that depend on internal state variables. Most of these macromechanical models are categorized as phenomenological models. The main differences among various phenomenological models are in the choice of internal state variables as well as in the evolution equations defining the thermomechanical driving forces (Brinson, 1993; Panico and Brinson, 2007; Arghavani et al., 2010; Saleeb et al., 2011; Chemisky et al., 2011; Zaki, 2012; Lagoudas et al., 2012; Andani et al., 2013). Some of these small-strain constitutive models are extended to large-scale and finite-strain models (Christ and Reese, 2009; Arghavani et al., 2011; Saleeb et al., 2013).

Asymmetry behavior in tension and compression is a well-know feature of shape memory alloys. In compression, the martensitic transformation from the austenitic phase is higher than in tension; maximum recoverable strain in compression is smaller than in tension; the hysteresis loop measured along the stress axis in compression is wider than in tension (Thamburaja and Nikabdullah, 2009). The difference of the hysteresis loops in tension and compression of the stress–strain response is due to different interaction energies and morphologies of the transformed phases for these loadings (Lim and McDowell, 1999). SMA models have been modified to capture these differences in tension and compression (Orgéas and Favier, 1998; Poorasadion et al., 2013).

Microplane theory is an efficient formulation in phenomenological models that describe complex material behaviors in a simple

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way. The rising interest in microplane modeling has led to its application to various materials such as quasi-brittle materials including concrete, soil, fiber composites, and stiff foams. Microplane formulation provides closed form relations for calculating the strain components in terms of the stress components. The other advantage of the approach is in the limited number of the required material parameters. These material parameters can be calculated in simple tension and torsion tests. The main feature of this modeling is in describing the behavior of a complex material with simple constitutive laws on each microplane. To this end, the approach finds the material behavior in any direction on each microplane and then uses the micro–macro homogenization process to obtain the overall macroscopic properties.

Various implementation methodologies exist (Bažant, 1984; Bažant and Prat, 1988a,b; Carol and Bažant, 1997). Carol and Prat (1990) used static constraint while Bažant and Caner (2005) used mixed static-kinematic constraint to implement microplane theory. In static constraint, the microscopic stresses on a specific microplane are equal to the projections of the macroscopic stress while in kinematic constraint the macroscopic strain tensor is projected on each microplane. The Normal–Tangential (N–T) split, by Bažant and Oh (1985), produces acceptable results in tensile loadings and has limitations in predicting the compression and shear loadings. To address these limitation, Carol et al. (1992, 2001, 2004), Bažant et al. (2000), and Kuhl et al. (2001) adopted a Volumetric–Deviatoric–Tangential (V–D–T) split, where the microplane normal strain and stress are divided into the volumetric and deviatoric components. Carol et al. (2001) showed that using the principle of complementary virtual work (PCVW) in a homogenization process for obtaining the overall macroscopic properties might violate the second principle of thermodynamics in certain loading conditions. In addition, they showed that some of the strain components that are used in the microplane level might not be conjugate with their stress counterparts. Leukart and Ramm (2002, 2003) and Leukart (2005) proposed a microplane model in thermodynamically-consistent framework with Volumetric–Deviatoric (V–D) split which can be viewed as a special case of the general V–D–T split. In this new split, the macroscopic strain tensor is projected into the normal and shear components and was shown that the new formulation in the strain components is an effective approach to remedy these deficiencies.

The first microplane modeling for SMAs was performed by Brocca et al. (2002). They divided shear stress on each microplane into two perpendicular components within the plane and used a 1D constitutive law for stress and strain components in normal and two shear directions on any arbitrary plane. They showed that some features such as stress–strain minor loops and tension–compression asymmetry could be predicted by the microplane model. Kadkhodaei et al. (2007, 2008) showed that microplane formulations with two shear directions on each plane have a directional bias nature and may result in prediction of unrealistic behaviors. Therefore, they utilized one resultant shear direction within each plane and proposed to use the Volumetric–Deviatoric split for normal direction. Mehrabi et al. (2012) and Mehrabi and Kadkhodaei (2013a) proposed a 3D phenomenological model based on the microplane theory in V–D–T split. They showed the capability of this approach in predicting martensite reorientation in multiaxial loadings.

Due to thermodynamic inconsistencies of the V–D–T split, there is a need for a more effective microplane formulation for SMAs. Therefore, in this work a microplane formulation based on V–D split in a thermodynamically-consistent framework is proposed. Within the context of these relations, Volumetric–Deviatoric components of the stresses in each microplane based on static constraint are presented. The new formulation based on the V–D split is compared numerically with the V–D–T split in uniaxial and pure torsion. The proposed model is also validated with experimental data.

This paper is organized as follows: in the second section, a standard thermodynamical procedure to obtain the microplane constitutive relations is summarized. Special focus of this section is on the microplane formulation extraction in a thermodynamic framework and motivation of this concept. The result of the new formulation in microplane model (V–D split) is compared with microplane formulation based on V–D–T split in Section 3. Finally, simulation results are compared with experimental results to assed the validity of the proposed model.

2. Microplane formulation based on thermodynamic approach

2.1. Thermodynamic derivation

Kadkhodaei et al. (2007, 2008), Mehrabi and Kadkhodaei (2013a) proposed a microplane model based on a static constraint in which a macroscopic stress tensor is projected on each plane. This leads to a decomposition of the stress vector into volumetric, deviatoric and tangential components, illustrated in Fig. 1. Macroscopic strain tensor based on microplane model (V–D–T split) derivative from principle of complementary virtual work (PCVW) is:

$$\boldsymbol{\varepsilon} = \varepsilon_V \mathbf{I} + \frac{3}{2\pi}\int_\Omega (\varepsilon_D \mathbf{N}) d\Omega + \frac{3}{2\pi}\int_\Omega (\boldsymbol{\varepsilon}_\mathbf{T} . \mathbf{T}) d\Omega \tag{1}$$

where

$$\varepsilon_V = \mathbf{V} : \boldsymbol{\varepsilon}, \varepsilon_D = \mathbf{D} : \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_\mathbf{T} = \mathbf{T} : \boldsymbol{\varepsilon}, \quad \text{and} \quad \varepsilon_N = \mathbf{N} : \boldsymbol{\varepsilon} \tag{2}$$

and

$$\mathbf{V} = \frac{\delta_{ij}}{3}, \mathbf{N} = \mathbf{V} + \mathbf{D} = n_i n_j, \mathbf{D} = n_i n_j - \frac{\delta_{ij}}{3}, \quad \text{and} \quad \mathbf{T} = T_{ijk} = \frac{1}{2}(n_i \delta_{jk} + n_j \delta_{ik} - 2 n_i n_j n_k) \tag{3}$$

Carol et al. (2001) showed that microplane formulation based on principle of complementary virtual work (PCVW) might violate the thermodynamic consistency in some loadings conditions. Therefore, Kuhl et al. (2001) as well as Leukart and Ramm (2002) proposed microplane formulations based on V–D split in a thermodynamically-consistent framework. The proposed formulation was based on a kinematic constraint in order to relate the macroscopic strain tensor to their microplane counterparts. Here, the procedure proposed by Carol et al. (2001), Kuhl et al. (2001) and Leukart and Ramm (2002) is revised based on a static constraint for shape memory alloys.

For the first step in a thermodynamically-consistent framework, a free energy $G^{mac}(\boldsymbol{\sigma}, \boldsymbol{k})$ is defined, where $\boldsymbol{k}$ is a set of internal variables. The macroscopic Gibbs free energy might be written as the integral of all microscopic free energies defined at the microplane level:

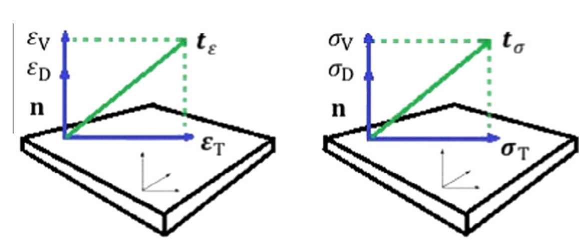

Fig. 1. The Volumetric–Deviatoric–Tangential microplane components of stress and strain.