International Journal of Solids and Structures 51 (2014) 2676–2686

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Vibration analysis of periodic cellular solids based on an effective couple-stress continuum model

Wenzheng Su [a,*], Shutian Liu [b]

[a] School of Civil and Safety Engineering, Dalian Jiaotong University, Dalian 116028, China
[b] State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, China

ARTICLE INFO

Article history:
Received 14 August 2013
Received in revised form 17 March 2014
Available online 8 April 2014

Keywords:
Cellular solids
Vibration
Couple-stress
Size effects

ABSTRACT

Cellular solids are usually treated as homogeneous continuums with effective properties. Nevertheless, these mechanical properties depend strongly on the ratio of the specimen size to the cell size. These size effects may be accounted for according to preliminary static analysis of effective continuums based on couple-stress theory. In this paper an effective dynamic continuum model, based on couple-stress theory, is proposed to analyze the behavior of free vibrations of periodic cellular solids. In this continuum model, the effective mechanical constants of the effective continuum are deduced by an equivalent energy method. The cellular solid structure is then replaced with the equivalent couple-stress continuum with same overall dimension and shape. Moreover, the finite element formulation of the couple-stress continuum for the generalized eigenvalue analysis is developed to implement the free vibration analysis. The eigenfrequencies of the effective continuum are then obtained via the shear beam theory or the finite element method. A conventional finite element analysis by discretizing each cell of the cellular solids is also carried out to serve as an exact solution. Several structural cases are calculated to demonstrate the accuracy and effectiveness of the proposed continuum model. Good agreement on structural eigenfrequencies between the effective continuum solutions and the exact solutions shows that the proposed continuum model can accurately simulate the dynamic behavior of the cellular solids.

1. Introduction

Cellular solids, such as metal foams, lattice truss materials and grid materials, are usually treated as homogeneous continuums with effective properties to simplify the analysis (Gibson and Ashby, 1997; Ashby et al., 2000). The primary purpose of such a model is to obtain the constitutive constants which depend on the effective density, the topology and even the size of the cell structure of the cellular solids. Generally, the unit cells in cellular solids are periodic. Moreover, if the unit cell is centrosymmetric, the effective continuum exhibits orthotropy. The properties of the effective continuum can be determined by tests. However, the classical homogenization method neglecting the gradient behavior of deformation is a cost-effective way to obtain these effective properties. The gradient behavior of deformation due to the material heterogeneity can be overlooked if the structural size is much larger than the cell structure of the cellular solids. Hence, the classical continuum model shows a high degree of accuracy in many situations. Up to now, several models have been proposed to determine the effective mechanical properties of cellular solids, such as elastic, plastic, buckling and thermal conductivity (Lakes, 1986; Gibson and Ashby, 1997; Deshpande et al., 2001; Wang and McDowell, 2004). More detailed work can also be referred in (Banerjee and Bhaskar, 2005) and references therein.

However, the effective mechanical properties of cellular solids show size-dependence if the dimension of a specimen or a structure is in a close order to the cell size. This behavior is called "size effects" in the literature (Onck et al., 2001; Tekoglu and Onck, 2008). Note that the local structural size may be in cell's order because the structure often has holes or other local sub-structures even if its global structural size is large. For example, a cellular beam or plate may have only several cells in its thickness direction with its length far larger than the cell size. The effective bending properties of such a structure exhibit size effects (Burgueno et al., 2005; Dai and Zhang, 2008; Tekoglu and Onck, 2008; Liu and Su, 2009; Su and Liu, 2010). It has been shown that this type of size effects can be predicted by the higher order theories, such as couple-stress theory, micropolar theory and strain gradient theory (Bigoni and Drugan, 2007; Liu and Su, 2009). Among these theories, the couple-stress theory (Mindlin, 1963) and the

* Corresponding author. Tel.: +86 411 84109342.
E-mail addresses: wzhsu@djtu.edu.cn (W. Su), stliu@dlut.edu.cn (S. Liu).

http://dx.doi.org/10.1016/j.ijsolstr.2014.03.043
0020-7683/

micropolar theory (Eringen, 1966), introduce additional rotational degrees of freedom besides the translational ones of particles, and contain intrinsic length scales in the constitutive relations. Hence, they can capture the higher order information of cell structures. Based upon this type of theories, the size effects of cellular solids in statics have been investigated in recent years (Tekoglu and Onck, 2008; Liu and Su, 2009).

Size effects are also found in the dynamic effective properties of cellular solids. Theoretically, the flexural vibration should also be size-dependent since the bending vibration is actually a time-dependent bending procedure. Similar to the static analysis of cellular solids, the size effects in dynamics can also be evaluated by higher order theories. In a related earlier work, Noor and Nemeth (1980) studied the eigenfrequencies and eigenmodes of a one-layer grid beam using micropolar theory and obtained accurate results. Banerjee and Bhaskar (2009) found that the eigenfrequencies of a cellular beam depend on the size ratio of the beam thickness to the cell length. They concluded that the classical effective continuum model can accurately simulate the real cellular beam's dynamic behavior only when the beam thickness is much larger than the cell size.

Although there are many researches on statics of cellular solids, the study on dynamics of cellular solids didn't receive relatively enough attention in the past (Baker et al., 1998; Wang and Stronge, 2001; Banerjee and Bhaskar, 2005, 2009). Note that Wang and Stronge (2001) investigated, by the use of micropolar theory, the dynamic responses of a regular hexagonal honeycomb. The object considered was treated as an infinite body (half-space) subjected to a harmonic concentrated force; and an analytically closed-form solution was presented. Nevertheless, for general elastic solids with finite region and/or complex cellular forms, it is difficult or impossible to obtain analytical solutions on the vibration problems. Moreover, the size effects become more important for finite size cellular solid structures. Hence, it is necessary to carry out further analysis on the dynamics of cellular solids by using numerical methods, such as Finite Element Method (FEM), based on the higher order continuum model.

In this paper, an effective continuum model based on couple-stress theory was proposed to study the free vibration behavior of cellular solid structures with finite size. The effective mechanical constants of cellular solids were firstly determined by the proposed effective continuum model. The eigenfrequencies of cellular structures were calculated using the proper beam theory or the FEM formulation of the effective couple-stress continuum. A conventional FEM analysis by discretizing each cell member of cellular solid was also carried out to serve as an exact solution to verify the accuracy of the proposed effective continuum model. Good agreement on structural eigenfrequencies between the continuum solutions and the exact solutions was found.

2. Effective couple-stress continuum model

2.1. Couple-stress theory

For a body in planar stress state, its displacement field can be given by $\boldsymbol{u} = (u, v, 0)^T$ and $\boldsymbol{\phi} = (0, 0, \phi)^T$. To simplify the analysis, it is assumed that there is no body force and body couple acting on the body, and damping within the body can be neglected. Accordingly, couple-stress theory yields the following governing differential equations of motion (Mindlin, 1963).

$$\begin{aligned} &\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} = \rho\frac{\partial^2 u}{\partial t^2}, \quad \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2} \\ &\frac{\partial m_{xz}}{\partial x} + \frac{\partial m_{yz}}{\partial y} + \tau_{xy} - \tau_{yx} = \Theta\frac{\partial^2\phi}{\partial t^2} \end{aligned} \tag{1}$$

where σ_x, σ_y, τ_{xy}, and τ_{yx} are the stress tensor components and m_{xz}, m_{yz} the couple-stress tensor components (Fig. 1), respectively. ρ is the material density and Θ the inertia of micro-rotation per unit volume.

Mindlin (1963) suggested resolving τ_{xy} and τ_{yx} into a symmetric part τ_S and an anti-symmetric part τ_A since the cross shear stress is not necessarily equal.

$$\tau_S = (\tau_{xy} + \tau_{yx})/2, \quad \tau_A = (\tau_{xy} - \tau_{yx})/2 \tag{2}$$

The symmetric part of the shear stress produces the usual shear strain while the anti-symmetric part tends to produce a local rigid rotation. For orthotropic solids, the constitutive equation can be expressed as follows.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_S \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} m_{xz} \\ m_{yz} \end{Bmatrix} = \begin{bmatrix} D_{11} & 0 \\ 0 & D_{22} \end{bmatrix} \begin{Bmatrix} \kappa_{xz} \\ \kappa_{yz} \end{Bmatrix} \tag{3}$$

or in a compact form

$$\boldsymbol{\sigma} = \boldsymbol{C\varepsilon}, \quad \boldsymbol{m} = \boldsymbol{D\kappa} \tag{4}$$

Consequently, the strain components ε_x, ε_x and γ_{xy}, as well as the curvature components κ_{xz} and κ_{yz} are defined as follows.

$$\begin{aligned} &\varepsilon_x = \partial u/\partial x, \quad \varepsilon_y = \partial v/\partial y, \quad \gamma_{xy} = \partial u/\partial y + \partial v/\partial x \\ &\kappa_{xz} = \partial\phi/\partial x, \quad \kappa_{yz} = \partial\phi/\partial y \end{aligned} \tag{5}$$

Note that the rotations are not independent but, rather, fully described by the displacement vectors in couple-stress theory.

$$\phi = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{6}$$

A crucial parameter in couple-stress theory is the characteristic length. This parameter describes the influence of couple-stress effects of the material. For orthotropic materials, there are four characteristic lengths (Bouyge et al., 2002; Liu and Su, 2009).

$$\begin{aligned} &l_{Gx} = \sqrt{B_{11}/G}, \quad l_{Ex} = \sqrt{2(1 + \nu_{21})B_{11}/E_{11}} \\ &l_{Gy} = \sqrt{B_{22}/G}, \quad l_{Ey} = \sqrt{2(1 + \nu_{12})B_{22}/E_{22}} \end{aligned} \tag{7}$$

where

$$\begin{aligned} &E_{11} = C_{11}(1 - \nu_{12}\nu_{21}), \quad E_{22} = C_{22}(1 - \nu_{12}\nu_{21}), \quad G = C_{66} \\ &\nu_{12} = C_{12}/C_{11}, \quad \nu_{21} = C_{21}/C_{22} \\ &B_{11} = D_{11}/4, \quad B_{22} = D_{22}/4 \end{aligned} \tag{8}$$

Note that the characteristic lengths vanish in the classical continuum.

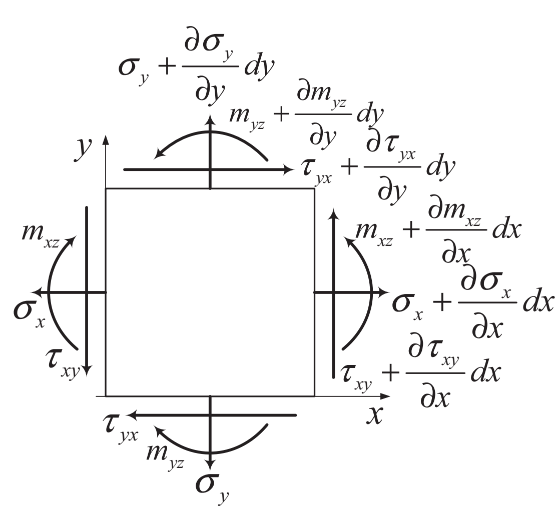

Fig. 1. Components of stress and couple-stress in a planar problem.