International Journal of Solids and Structures 51 (2014) 2706–2712

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Adhesion between elastic cylinders based on the double-Hertz model

Fan Jin [a,b], Wei Zhang [a], Sulin Zhang [c], Xu Guo [a,*]

[a] State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian 116023, PR China
[b] Institute of Systems Engineering, China Academy of Engineering Physics, Mianyang, Sichuan 621900, PR China
[c] Department of Engineering Science and Mechanics, Department of Materials Science and Engineering, The Pennsylvania State University, University Park, PA 16802, United States

ARTICLE INFO

Article history:
Received 10 November 2013
Received in revised form 29 March 2014
Available online 13 April 2014

Keywords:
Contact mechanics
Adhesion
Cohesive zone model
Double-Hertz model
Surface energy

ABSTRACT

A cohesive zone model for two-dimensional adhesive contact between elastic cylinders is developed by extending the double-Hertz model of Greenwood and Johnson (1998). In this model, the adhesive force within the cohesive zone is described by the difference between two Hertzian pressure distributions of different contact widths. Closed-form analytical solutions are obtained for the interfacial traction, deformation field and the equilibrium relation among applied load, contact half-width and the size of cohesive zone. Based on these results, a complete transition between the JKR and the Hertz type contact models is captured by defining a dimensionless transition parameter μ, which governs the range of applicability of different models. The proposed model and the corresponding analytical results can serve as an alternative cohesive zone solution to the two-dimensional adhesive cylindrical contact.

1. Introduction

Adhesive forces that act between contacting bodies play a key role in determining the mechanical behavior of small-scale systems. For instance, adhesive force can induce significant local stress in atomic force microscopy (AFM) which can therefore result in substantial wear and tip degradation (Liu et al., 2010). With increasing usage of micro-scale components and devices, it is imperative to obtain a better understanding of the contact behavior considering adhesive forces.

Since Hertz's seminal work (1882) on the unilateral contact of elastic spheres, numerous studies have been conducted on the adherence of spherical bodies. Bradley (1932) examined the attractive force between two rigid spheres by considering the molecular interactions. Later on, two famous models for adhesive contact between elastic spheres were proposed by Johnson et al. (1971) (JKR model) and Derjaguin et al. (1975) (DMT model), respectively. However, the magnitudes of the pull-off force predicted by the JKR and DMT models are quite different. Tabor (1977) then compared the two models and showed that JKR and DMT models represent two limiting cases of adhesive contact and their ranges of validity can be assessed by a dimensionless parameter (i.e., Tabor parameter) (Greenwood, 1997; Johnson and Greenwood, 1997; Barthel, 2008). To be more specific, the JKR model works well for soft materials with relatively high surface energy while the DMT model is more appropriate for hard solids with low surface energy. The first cohesive zone model which can allow for the transition between the JKR and DMT models was established by Maugis (1992). In this model (the so-called Maugis–Dugdale (M–D) model), the adhesive stress acting over the cohesive zone is assumed to be constant (i.e., Dugdale (1960)), which facilitates the derivation of analytical solutions. Soon afterwards, this model was also extended to describe the noncontact case (Kim et al., 1998).

In parallel with the M–D model, Greenwood and Johnson (1998) put forward an alternative cohesive zone model, known as the double-Hertz (D-H) model, which is also applicable to arbitrary values of Tabor parameter. In this model, the adhesive force within the cohesive zone is described by the difference between two Hertzian pressure distributions of different contact radii. It was found that results obtained by the D-H model are very close to those from the M–D model. However, the D-H model is more analytically tractable than the M–D model since the corresponding analysis relies solely on the classical Hertzian solutions. For this reason, the D-H model is often adopted to study the adhesion behavior of complex contact systems involving rough contact surfaces (Persson, 2002; Zhang et al., 2014), viscoelastic materials (Haiat et al., 2003) and functionally graded elastic solids (Jin et al., 2013). Recently, the D-H model was reconsidered in a slightly different context using an auxiliary function method (Barthel, 2012).

The above advances in contact mechanics of three-dimensional spherical bodies laid a solid foundation for the study of two-dimensional cylindrical contact systems. Barquins (1988) developed the

* Corresponding author. Tel.: +86 411 84707807.
E-mail address: guoxu@dlut.edu.cn (X. Guo).

http://dx.doi.org/10.1016/j.ijsolstr.2014.04.003
0020-7683/

JKR-type solutions for elastic cylinders and verified it experimentally. With use of Barquins's theory, Chaudhury et al. (1996) predicted the surface and adhesion energies of elastomeric polydimethylsiloxane (PDMS) successfully. The two-dimensional JKR model was also extended to the non-slipping case with the frictionless contact assumption relaxed (Chen and Gao, 2006a; 2007) and the conforming contact case with the half-plane assumption relaxed (Sundaram et al., 2012).

The above mentioned JKR-based models, however, do not consider the adhesion forces outside the contact area and therefore are only applicable to soft bodies with relatively large Tabor parameters. For general material properties, Baney and Hui (1997) proposed the first cohesive zone model for cylindrical contact in the framework of M–D model, Morrow and Lovell (2005) then extended Baney and Hui's theory to the case where the surfaces are not within intimate contact but are within the range of adhesive interaction. The same two-dimensional M–D analysis was also performed by Johnson and Greenwood (2008) independently, with emphasis on the pull-off force. Chen and Gao (2006b) presented an analogous M-D model of a cylinder in non-slipping adhesive contact with a stretched substrate. Furthermore, based on the two-dimensional M-D model, Sari et al. (2005) also investigated the sliding and rolling motion of a cylinder on the substrate subjected to combined normal and tangential forces.

The present study is aimed to extend the three-dimensional double-Hertz model of Greenwood and Johnson (1998) to a plane strain problem, with emphasis on establishing a set of simple analytical solutions which are applicable for a full range of Tabor parameters. These solutions can not only describe a complete transition between the two-dimensional JKR and the Hertz type contact models, but also exhibit as equally effective as the two-dimensional M–D model.

The rest of the paper is organized as follows. We first extend the double-Hertz model to the cylindrical contact system in Section 2. The main analytical results are then presented in dimensionless form in Section 3. Section 4 discusses the reduction of the proposed model in two limiting cases of small and large cohesive zones. The traction-separation relation within the cohesive zone is examined in Section 5. Finally, some concluding remarks are provided in Section 6.

2. Two-dimensional double-Hertz model

Fig. 1a shows the adhesive contact between two dissimilar elastic cylinders with parallel axes under a prescribed load P (with unit N/m and negative when tensile). Contact occurs over a rectangular region of width $2a$. In fact, if the tangential tractions are neglected, this problem is equivalent to the plain strain frictionless contact problem between a rigid cylinder of radius R and an elastic half-plane with a effective Young's modulus E^*, where

$$1/R = 1/R_1 + 1/R_2 \tag{2.1}$$

and

$$1/E^* = (1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2, \tag{2.2}$$

respectively. In Eqs. (2.1) and (2.2), R_1, R_2 are the radii, ν_1, ν_2 are the Poisson radios and E_1, E_2 are the Young's moduli of the contacting cylinders, respectively (Johnson, 1985). For subsequent analytical treatment, as shown in Fig. 1b, a Cartesian coordinate system (x, z) is set up with origin at the center of the contact zone and z direction pointing into the half-plane. The distribution of surface traction consists of two terms: the Hertz pressure p_H acting on a contact region of width $2a$ and the adhesive tension p_A acting on an interaction zone of width $2c$. The noncontact regions bounded by half-widths a and c (i.e., $a \leqslant |x| \leqslant c, z = 0$) are known as the cohesive

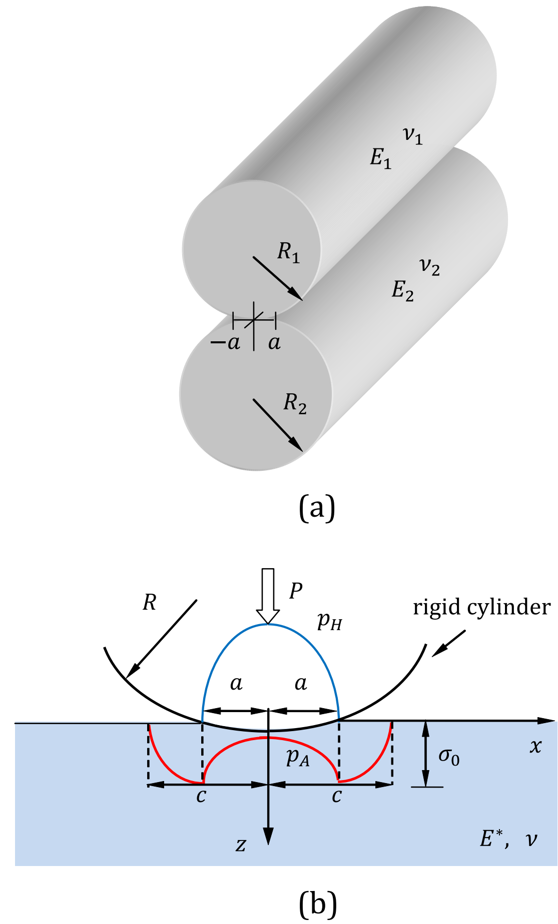

Fig. 1. (a) Schematics of adhesive contact between two elastic cylinders. The constants (E_1, ν_1) and (E_2, ν_2) denote Young's moduli and Poisson's ratios of the two cylinders. (b) A rigid cylinder in frictionless adhesive contact with an elastic half-plane under a normal load P (negative when tensile). The distribution of surface traction consists of two terms: the Hertz pressure p_H acting on the contact zone of width $2a$ and an adhesive traction p_A acting on the interaction zone of width $2c$.

zones. Since the present problem is symmetry with respect to the z-axis, we only quote the equations for $x \geqslant 0$ in the following analysis.

In the absence of adhesive force, the Hertz-type pressure distribution between a rigid cylinder and an elastic half-plane is given by (Johnson, 1985)

$$p(x) = \frac{E^*}{2R}(a^2 - x^2)^{1/2}, \quad |x| \leqslant a \tag{2.3}$$

which corresponds to a prescribed load

$$P = \frac{\pi a^2 E^*}{4R} \tag{2.4}$$

The derivative of the surface normal displacement with respect to x can be expressed as

$$\frac{\partial u_z}{\partial x} = -\frac{x}{R}, \quad 0 \leqslant x \leqslant a, \tag{2.5a}$$

$$\frac{\partial u_z}{\partial x} = -\frac{2}{\pi E^*}\int_{-a}^{a}\frac{p(s)}{x-s}\mathrm{d}s = -\frac{x-\sqrt{x^2-a^2}}{R}, \quad x \geqslant a, \tag{2.5b}$$

According to Greenwood and Johnson (1998), the essential idea behind the proposed two-dimensional double-Hertz model is to represent the adhesive tensile traction by resorting to the difference of two Hertzian pressure distributions, that is,