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Low-energy impact response of composite and sandwich composite plates with piezoelectric sensory layers

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ABSTRACT

An efficient model reduction based methodology is presented for predicting the global (impact force, plate deflection and electric potential) and through-thickness local (interfacial strains and stresses) dynamic response of pristine simply-supported cross-ply composite and sandwich composite plates with piezoelectric sensory layers subjected to low-energy impact. The through-thickness response of the laminate is modelled using coupled higher-order layerwise displacement-based piezoelectric laminate theories. Linearized contact laws are implemented for simulating the impactor–target interaction during impact. The stiffness, mass, piezoelectric and permittivity matrices of the plate are formulated from ply to structural level and reduced by applying a Guyan reduction technique to yield the structural system in state space. This reduction technique enables the formulation of a plate–impactor structural system of minimum size (1 term per vibration mode for composite plates – 2 terms for sandwich plates) and reduces computational cost, thus facilitating applicability for real-time impact and vibration control.

1. Introduction

The dynamic response of composite and sandwich composite plates under low-energy impact is of practical significance in automotive, aerospace and every-day life applications, in cases such as a tool drop during repair, a hit by a runaway stone or the fall of a composite cell-phone case from a table to the floor, where the damage caused may be invisible, while the inclusion of piezoelectric sensory layers into the lamination enables monitoring of the structural response on-site in real-time. The prediction of the global and local through-thickness impact response of such smart structures is essential during the design phase in order to determine the impact force, the type and duration of impact, to estimate stresses at the interface between composite and piezoelectric material layers and to quantify the signals acquired by the piezoelectric layers. These predictions, combined with a computationally efficient plate–impactor system model, are expected to contribute to the development of appropriate algorithms for active impact control and control for energy harvesting.

The importance of the impact response of composite and sandwich composite structures is highlighted by the amount of work conducted in this field so far. Extensive related literature reviews have been conducted among others by Cantwell and Morton (1991), Abrate (1997, 1998, 2001), Qiu and Yu (2011) and Chai and Zhu (2011), while relevant papers have been reported by Stronge (2000) in his book on impact mechanics. On the basis of the kinematic assumptions used to predict the response of the impacted composite or sandwich composite structure, the existing models may be divided into two main categories: (i) mass–spring models without (Shivakumar et al., 1985; Wu and Yu, 2001; Olsson, 2002; Zhou and Stronge, 2006) or with dampers (Olsson, 2003; Anderson, 2005) and (ii) full continuum models based on energy equilibrium equations. The latter may encompass exact, analytical or finite element solutions and can potentially predict the impact response in several positions of the structure, including all displacements, strains and stresses. Moreover, depending on the amount of the vibration modes taken into account, continuum models can capture multiple impacts caused by the induced vibration triggered by the impact event. Analytical solutions for composite plates subjected to low-velocity impacts have been developed among others by Christoforou and Swanson (1991), and Christoforou and Yigit (1998), on the basis of Kirchhoff's plate theory kinematics and a linearized elasto-plastic contact law between the plate and impactor (Yigit and Christoforou, 1994), whereas Chun and Lam (1998) implemented Reddy's higher-order single layer plate theory and a Hertzian contact law. Finite element solutions for predicting the low-velocity impact response of composite plates have been reported among others by Sun and Chen (1985), who developed a quadratic Lagrange element based on Reissner–Mindlin kinematics and an experimentally determined non-linear indentation law, Wu and Chang (1989), who formulated

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a 3-D plate theory and a corresponding brick element to study out of plane stresses in addition to plate deflection and contact force, and Choi and Chang (1992), who predicted damage due to impact and reported relevant experimental results. As far as sandwich plates subjected to low-velocity impact are concerned, Palazotto et al. (2000) formulated a C^2-continuous finite element based on higher-order single-layer kinematics and geometrical nonlinearity for conducting progressive failure analysis, whereas Besant et al. (2001) combined first order shear shell elements for the faces with brick elements for the core, considered elastoplastic material behaviour and degradation, and reported numerical and experimental results. Yang and Qiao (2005) developed an analytical solution based on a higher-order laminate theory and predicted global (contact force and deflection) and local through-thickness (propagation of normal and shear stresses) response, and predicted failure locations, time and modes in sandwich composite beams. Icardi and Ferrero (2009) reported a refined plate element based on global-local 3-D layerwise kinematic assumptions, considered material degradation and predicted damage and through-thickness distributions of transverse displacement and interlaminar shear stress, in addition to temporal variation of the contact force. A Ritz-type solution based on 3-D higher-order single-layer mixed kinematics was developed by Malekzadeh et al. (2006) for predicting the dynamic response of sandwich plates subjected to multiple impacts. Analytical and finite element solutions were also reported by Hoo Fatt and Park (2001) and Kärger et al. (2008), respectively. Experimental results for composite and sandwich composite plates subjected to low-velocity impact were reported among others by Sjöblom et al. (1988), Lee et al. (1993), Ambur et al. (1995), Hazizan and Cantwell (2002), Schubel et al. (2005), Christoforou et al. (2010) and Yang et al. (2013). The idea of embedding piezoelectric sensors to composite structures in order to detect impact location and reconstruct the contact force time-profile was reported in the late 90's by Tracy and Chang (1998) and Seydel and Chang (2001), and has been elaborated for the design of real-time monitoring networks (Park et al., 2009; Liu and Chattopadhyay, 2013). The active control of impact response of composite plates and shells by means of piezoelectric layers and patches towards the minimization of contact force has been studied by Saravanos and Christoforou (2002a,b), who developed an analytical solution based on first-order shear kinematics for the composite laminate and a linear layerwise approximation of electric potential. Yet, the local through-thickness impact response of sandwich piezoelectric composite plates in the case of low-velocity and low-energy impact has not been studied so far. Moreover, in the vast majority of existing work employing full continuum models for predicting impact response of composite or sandwich plates, the full plate–impactor system is solved. This leads to large matrix sizes and increased computational effort for detailed through-thickness modelling, such as in the case of layerwise laminate theories.

In this paper, an efficient Ritz-type solution is presented, which is based on higher-order layerwise through-thickness kinematic assumptions and a Guyan reduction technique, for predicting both global (plate deflection, contact force and electric potential) and local (through-thickness distribution of displacements, strains and stresses) response of pristine simply-supported cross-ply composite and sandwich composite plates with piezoelectric layers subjected to low-energy impact. In the proposed method, the full structural system containing all Fourier modal displacement amplitudes is reduced to one containing a single deflection amplitude per mode, leading to dramatic savings in size and computational effort, which is a most useful capability for real-time control applications. Still, the information included in the full structural system is retained and recovered after solving the reduced system by expressing the dependent modal variables and their derivatives via the modal deflection of the plate. The accuracy of the proposed method is validated by comparisons with published numerical results for composite plates without/with piezoelectric layers and with experimental results for sandwich composite plates.

2. Theoretical formulation

In this section, the integrated theoretical framework developed for simulating the impact response of a sandwich composite plate with piezoelectric layers subjected to low-energy impact is developed, starting from a general composite material ply with piezoelectric properties and arriving to the solution of the structural system in state-space.

2.1. Formulation of plate-subsystem structural matrices

2.1.1. Basic physical assumptions

The theoretical framework developed is based on the following assumptions:

- The impact energy is low, such as no material damage is induced by the impact event.
- The impact is elastic, thus, there is no loss of energy in the form of heat.
- The laminate plies are perfectly bonded together throughout the impact event.

2.1.2. Governing material equations

In general, the laminate layers including the piezoelectric, composite and foam plies are assumed to exhibit linear piezoelectric behaviour. In the following formulation, displacements and electric potential and all other variables arising from these (strains, stresses, etc.) are time-dependent. The ply constitutive equations in the natural coordinate system Oxyz (Fig. 1(a)) have the form:

$$\begin{aligned} \boldsymbol{\sigma}_i &= C^E_{ij}\mathbf{S}_j - (e_{mi})^T\mathbf{E}_m \\ \mathbf{D}_m &= e_{mj}\mathbf{S}_j + \varepsilon^S_{mm}\mathbf{E}_m \end{aligned} \quad (1)$$

where i, $j = 1,\ldots,6$ and $m = 1,\ldots,3$; $\boldsymbol{\sigma}_i$ and $\mathbf{S}_j$ are the mechanical stress and engineering strain, respectively, in vectorial notation; $\mathbf{E}_m$ is the electric field vector; $\mathbf{D}_m$ is the electric displacement vector; C_{ij} is the elastic stiffness tensor; e_{mj} is the piezoelectric tensor arising from the piezoelectric charge tensor and the stiffness tensor; and ε_{mm} is the electric permittivity tensor of the material. The form of the above tensors is shown in Appendix A. Superscripts E and S indicate a constant electric field, and strain conditions, respectively. The above equations may encompass the behaviour of both an off-axis homogenized fibrous piezoelectric ply and a passive composite ply ($e_{mj} = 0$). The electric field vector E_m is the gradient of the electric potential $\boldsymbol{\varphi}$ along basis vectors x, y, z of the natural coordinate system:

$$\mathbf{E}_m = -\partial\boldsymbol{\varphi}_m/\partial\mathbf{x}_m \quad (2)$$

In the current work, piezoelectric components polarized through-thickness are considered.

2.1.3. Through-thickness kinematic assumptions

A typical composite or sandwich composite laminate with piezoelectric components is subdivided into n discrete layers as shown schematically in Fig. 1(a). Each discrete-layer may contain either a single ply, a sub-laminate, or a sub-ply. In the case of composite plates, the displacement field assumed through the thickness of the laminate is based on a 2-D higher-order layerwise formulation (HLPT 2-D – Plagianakos and Saravanos, 2008), which approximates displacements and electric potential by piecewise linear, parabolic and cubic functions of the discrete layer thickness (Fig. 1(b)), while maintaining displacement continuity across