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Some half-space problems of cubic piezoelectric materials

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ABSTRACT

Half-space problems of a cubic piezoelectric material subjected to anti-plane deformation and in-plane electric field are studied. A general solution in terms of the integration of the boundary data prescribed over the surface of the semi-infinite domain is derived. Based on the general solution, the problem of a concentrated line force acting on the surface is treated and ensuing electromechanical response is determined. The solution to the problem of a screw dislocation in the half-space is also obtained, and the result is exploited to study a sub-surface crack problem by simulating the crack as a continuous distribution of dislocations.

1. Introduction

Piezoelectric materials are widely used in various fields of engineering, ranging from simple pressure transducers and motion accelerometers to complicated integration systems such as smart structures and microelectromechanical systems (MEMS). They can act simultaneously as sensors and actuators, and therefore may encounter complex electromechanical loadings. Thus, it is important to understand how these materials response to externally applied loads. Defects in them play a major role in determining the strength and reliability of devices made of these materials. Intensive research has resulted in numerous articles on this topic and interested readers are referred to review articles for a long list of references (Zhang et al., 2002; Zhang and Gao, 2004; Kuna, 2010). After perusing these publications, it is found that most of exact and explicit solutions are restricted to the class with hexagonal symmetry; few explicit results are available for other classes. Recently attempts have been made to fill this void of information by solving some defect related problems in cubic piezoelectric crystals. The piezoelectric potential induced by a screw dislocation in cubic crystals was analyzed in Chiang (2012) and a more complete account was given in Chiang (2013a). Crack problems were solved in Chiang (2013b) where some useful solutions have been obtained. To continue the pursuit of this goal, analytic solutions of some half-space problems of cubic piezoelectric crystals are derived in this article. The present results are exact and explicit, and therefore may shed some light on this complicated problem.

The paper is organized as follows. In the next section, the governing equations for the anti-plane deformation coupled with in-plane electric field in a cubic piezoelectric medium are briefly reviewed. In Section 3, by the method of analytic function of complex variables, solutions of half-space problems are derived for two electric boundary conditions specified on the surface of the semi-infinite region. To illustrate the general solution, a concentrated line force acting on the surface is specifically treated and ensuing electromechanical fields are explicitly shown in Section 4. The fundamental solution concerning a screw dislocation beneath the free surface is also solved in this section. In Section 5, a sub-surface crack problem is solved by simulating the crack as a continuous distribution of dislocations.

2. Governing equations and general solutions

Since the material has a cubic symmetry, it is advantageous to take a coordinate system as shown in Fig.1 that the x-axis is in the [1 0 0] direction and the y-axis point to [0 1 0] direction. The problem under consideration is an out-of-plane shear coupled with in-plane electric field; as shown earlier that for the present problem the displacement w in the direction of z-axis and the electric potential Φ must satisfy (Chiang, 2012, 2013a,b)

$$C_{44}\frac{\partial^2 w}{\partial y^2} + C_{44}\frac{\partial^2 w}{\partial x^2} + 2e_{14}\frac{\partial^2 \Phi}{\partial x \partial y} = 0 \qquad (1)$$

$$2e_{14}\frac{\partial^2 w}{\partial x \partial y} - \kappa_{11}\frac{\partial^2 \Phi}{\partial x^2} - \kappa_{11}\frac{\partial^2 \Phi}{\partial y^2} = 0 \qquad (2)$$

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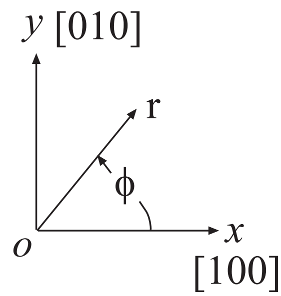

Fig. 1. Rectangular coordinates and polar coordinates.

where C_{44}, e_{14} and κ_{11} are the elastic constant, piezoelectric constant and permittivity of the material. By eliminating Φ from these two equations, it is found that

$$\frac{\partial^4 w}{\partial x^4} + \left(2 + \frac{4e_{14}^2}{C_{44}\kappa_{11}}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = 0 \tag{3}$$

If the displacement $w(x, y)$ is assumed to have the form of $w(x + \mu y)$, then μ must satisfy the following characteristic equation

$$\mu^4 + 2(1 + 2h)\mu^2 + 1 = 0 \tag{4}$$

The four roots of Eq. (4) are all pure imaginary and can be written as follows: $\mu_1 = is_1 = i\left(\sqrt{1+h} + \sqrt{h}\right)$, $\mu_2 = is_2 = i\left(\sqrt{1+h} - \sqrt{h}\right)$, $\mu_3 = -\mu_1$ and $\mu_4 = -\mu_2$ with $i = \sqrt{-1}$ and $h = e_{14}^2/(C_{44}\kappa_{11})$. It is noted that $\mu_1\mu_2 = -1$ (or $s_1 s_2 = 1$).

Since the displacement must be a real function, it is concluded that the general solution of Eq. (3) is

$$w = 2\mathrm{Re}[F_1(z_1) + F_2(z_2)] \tag{5}$$

where Re[] denotes the real part of a complex function; F_1 and F_2 are some arbitrary functions with $z_1 = x + \mu_1 y$ and $z_2 = x + \mu_2 y$. On the other hand, since the electric potential Φ must satisfy Eqs. (1) and (2), it is concluded that

$$\Phi_k'' = \lambda_k F_k'', \quad k = 1, 2 \tag{6}$$

where $\lambda_1 = -\lambda_2 = i\lambda$ and $\lambda = -\sqrt{C_{44}/\kappa_{11}}$. Thus, the electric potential can be written as

$$\Phi = 2\mathrm{Re}[\lambda_1 F_1(z_1) + \lambda_2 F_2(z_2) + a_1 z_1 + a_2 z_2 + b] \tag{7}$$

Eqs. (5) and (7) are the general solution that satisfy the governing equations of the problem. Furthermore, when the displacement and the electric potential of a specific problem have been determined, the electric field, stress and electric displacement can be found by the following equations

$$E_x = -2\mathrm{Re}\left[\lambda_1 F_1'(z_1) + \lambda_2 F_2'(z_2)\right] \tag{8}$$

$$E_y = -2\mathrm{Re}\left[\lambda_1 \mu_1 F_1'(z_1) + \lambda_2 \mu_2 F_2'(z_2)\right] \tag{9}$$

$$\tau_{zx} = 2\mathrm{Re}\left[(C_{44} + e_{14}\lambda_1\mu_1)F_1'(z_1) + (C_{44} + e_{14}\lambda_2\mu_2)F_2'(z_2)\right] \tag{10}$$

$$\tau_{yz} = 2\mathrm{Re}\left[(C_{44}\mu_1 + e_{14}\lambda_1)F_1'(z_1) + (C_{44}\mu_2 + e_{14}\lambda_2)F_2'(z_2)\right] \tag{11}$$

$$D_x = 2\mathrm{Re}\left[(e_{14}\mu_1 - \kappa_{11}\lambda_1)F_1'(z_1) + (e_{14}\mu_2 - \kappa_{11}\lambda_2)F_2'(z_2)\right] \tag{12}$$

$$D_y = 2\mathrm{Re}\left[(e_{14} - \kappa_{11}\lambda_1\mu_1)F_1'(z_1) + (e_{14} - \kappa_{11}\lambda_2\mu_2)F_2'(z_2)\right] \tag{13}$$

From these equations, it can be seen that the influence of the material constants on the solution is through the complex parameters μ_1, μ_2, λ_1 and λ_2. The material constants and associated complex parameters are shown in Tables 1 and 2 for two typical cubic piezoelectric crystals: Bismuth Germanate and Bismuth Germanium Oxide (Auld, 1973).

Table 1
Material constants and electromechanical coupling parameter h of Bismuth Germanate ($Bi_4Ge_3O_{12}$) and Bismuth Germanium Oxide ($Bi_{12}GeO_{20}$).

Material	C_{44} (10^{10} N/m^2)	e_{14} (C/m^2)	κ_{11}(10^{-12} F/m)	h
$Bi_4Ge_3O_{12}$	4.36	0.0376	142	0.00022835
$Bi_{12}GeO_{20}$	2.55	0.99	336	0.114391

Table 2
Material parameters of Bismuth Germanate and Bismuth Germanium Oxide. Note that $\mu_1 = is_1$, $\mu_2 = is_2$, $\lambda_1 = i\lambda$ and $\lambda_2 = -i\lambda$.

Material	s_1	s_2	λ
$Bi_4Ge_3O_{12}$	1.0152254	0.9850029	-1.75226×10^{10}
$Bi_{12}GeO_{20}$	1.3938641	0.7174300	-8.71165×10^9

3. Solutions to half-space problems

Consider a half-space domain which occupies the lower part of $y = 0$ surface in the coordinates as shown in Fig. 1. A variety of boundary conditions can be specified on the surface. In this section the solutions are given for the following two types of boundary conditions: on $y = 0$, (A) $\tau_{yz} = S^*(x)$, $\Phi = \Phi^*(x)$, and (B) $\tau_{yz} = S^*(x)$, $D_y = D^*(x)$ as shown in Fig. 2. Electric boundary condition (A) is intended for the case that the electric potential across the surface of piezoelectric crystals can be imposed or measured. On the other hand the electric boundary condition (B) is intended for applications where the surface is in contact with conductors of known charge density or in contact with dielectrics of known electric induction normal to the surface.

Before deriving the solution, it is first noted that for an analytic function f which vanishes at infinity has the following property,

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{f(\xi)}{\xi - z} = -f(z), \quad \frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\overline{f(\xi)}}{\xi - z} = 0 \tag{14}$$

where z is in the lower half-space ($y \leqslant 0$), and $\overline{f(z)} = \bar{f}(\bar{z})$. A bar over a variable or a function denotes its complex conjugate. The following derivation of the solution is parallel to that in Lekhnitskii (1968) which is a generalization of the method used in Muskhelishvili (1963).

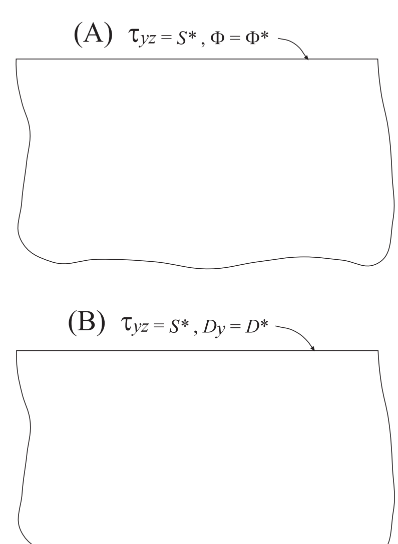

Fig. 2. Two types of boundary conditions imposed on the surface of semi-infinite region.