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Statistical model of nearly complete elastic rough surface contact

Y. Xu, R.L. Jackson *, D.B. Marghitu

Department of Mechanical Engineering, Auburn University, Auburn, AL 36849, United States

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ABSTRACT

In the area of homogeneous, isotropic, linear elastic rough surface normal contact, many classic statistical models have been developed which are only valid in the *early contact* when real area of contact is infinitesimally small, e.g., the Greenwood–Williamson (GW) model. In this article, newly developed statistical models, built under the framework of the (i) GW, (ii) Nayak–Bush and (iii) Greenwood's simplified elliptic models, extend the range of application of the classic statistical models to the case of *nearly complete contact*. Nearly complete contact is the stage when the ratio of the real area of contact to the nominal contact area approaches unity. At nearly complete contact, the non-contact area consists of a finite number of the non-contact regions (over a finite nominal contact area). Each non-contact region is treated as a mode-I "crack". The area of each non-contact region and the corresponding trapped volume within each non-contact region are determined by the analytical solutions in the linear elastic fracture mechanics, respectively. For a certain average contact pressure, not only can the real area of contact be determined by the newly developed statistical models, but also the average interfacial gap. Rough surface is restricted to the geometrically-isotropic surface, i.e., the corresponding statistical parameters are independent of the direction of measurement. Relations between the average contact pressure, non-contact area and average interfacial gap for different combinations of statistical parameters are compared between newly developed statistical models. The relations between non-contact area and average contact pressure predicted by the current models are also compared with that by Persson's theory of contact. The analogies between the classic statistical models and the newly developed models are also explored.

1. Introduction

Elastic rough surface contact models have been developed for more than 50 years since the first one created by Archard (1957). Because of the complexity of the boundary conditions on the contact interfaces, i.e., the surface traction distribution and surface displacement field, the elastic rough surface contact problem cannot be completely solved analytically except for the case when contact becomes complete.[1] The statistical based model is one of the approximate models and was first introduced by Greenwood and Williamson (1966). This is the first model combining the random process with the elastic contact model (Hertzian spherical contact model). Nayak, 1971 modeled the rough surface as a two-dimensional (2D) isotropic, Gaussian, random process, which is referred to as *Nayak's random theory*. Bush and Thomas, 1982 applied Nayak's random theory in the elastic rough surface contact model (Nayak–Bush model) by assuming that the asperities are axisymmetric. Bush et al., 1975 developed, up till now, the most complete statistical model (BGT model) based on Nayak's random theory. Utilization of the Hertzian elliptic contact model complicates the BGT model. Greenwood, 2006 reduced the complexity of the BGT model by introducing an mildly Hertzian elliptic contact model which is only valid for the elliptic asperities with similar principle curvatures. This model is referred to as *Greenwood's simplified elliptic model*. A good agreement can be found between the BGT model and Greenwood's simplified elliptic model (Greenwood, 2006). Those statistical models, discussed above, are now referred to as the *classic* statistical models. One of the main assumptions adopted in the classic statistical models is that the interactions between the neighboring contacting asperities, due to the elasticity of the substrate, are ignored, which limits the application of the classic statistical models within the light load (real area of contact $\ll$ nominal contact area) range.

Nearly all the newly developed statistical models (Bush et al., 1976; O'Callaghan and Cameron, 1976; Francis, 1977; McCool and Gassel, 1981) after the Greenwood and Williamson (GW) model restrict their application within the case of *early contact* where the real area of contact is infinitesimally small. Few attempts have been made to introduce the asperity interaction (equivalently, the elasticity of the substrate) in the classic statistical model (Zhao and Chang, 2001, e.g., Ciavarella et al., 2008). *Nearly complete contact* is defined as the stage where isolated *non-contact regions* (easily visualized as "islands") of infinitesimally small areas are surrounded by the

* Corresponding author. Tel.: +1 334 844 3340.
E-mail addresses: yang.xu@auburn.edu (Y. Xu), robert.jackson@eng.auburn.edu (R.L. Jackson), MARGHDB@auburn.edu (D.B. Marghitu).

[1] Here we implicitly assume that the geometry of the rough surface is *deterministic*.

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Nomenclature

A	real area of contact, i.e., size of domain Ω_c
A^*	contact ratio, $A^* = A/A_n$
A_n	nominal contact area, i.e., the size of Ω
C	rigid body displacement on the plane $z = 0$ of an elastic half-space
$C(r)$	circumference of an ellipse
C_1, C_2	constants, $C_1 = \alpha/(2\alpha - 3)$ and $C_2 = C_1\left(\frac{12}{\alpha}\right)^{1/2}$
E	Young's modulus of equivalent rough surface
$E[\bullet]$	average value of the process inside the square bracket
E^*	effective material modulus, $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$
$E_i, i = 1,2$	Young's modulus of rough surface 1 and 2
K_I	stress intensity factor of a mode-I crack
P	total contact load over the domain Ω
P^*	dimensionless contact load at early contact, $P^* = P/(E^* A_n)$
R	average radius of curvature of the asperities
S	power spectrum density of a rough surface
V	variance of the "pressure surface", $p = p_c(x,y)$, $V = \sqrt{m_0^p}$
Ω	nominal contact domain
Ω_c, Ω_{nc}	contact and non-contact domains
Φ	probability density function of the asperity height of a rough surface (or "pressure surface")
α	bandwidth parameter: $\alpha = \frac{m_0 m_4}{m_2^2}$
α_1^p	dimensionless parameter $\alpha_1^p = \frac{m_0^p}{\sqrt{m_2^p}} \frac{1}{E^* \sigma}$
$\overline{A}$	non-contact area, i.e., size of domain Ω_{nc}
$\overline{A}^*$	non-contact ratio, i.e., $\overline{A}^* = 1 - A^*$
$\overline{A}_n$	area of each non-contact region
$\overline{V}_i$	trapped volume within a non-contact region
$\bar{g}$	average interfacial gap
$\bar{g}^*$	dimensionless average interfacial gap, $\bar{g}^* = \bar{g}/\sigma$
$\bar{p}$	average pressure over the domain Ω
$\bar{p}^*$	dimensionless average pressure, $\bar{p}^* = \bar{p}/\sigma_s^p$ in modified GW model and $\bar{p}^* = \bar{p}/\sqrt{m_0^p}$ in modified Nayak–Bush and Greenwood's simplified model
$\bar{p}_c$	critical value of the average pressure across which rough contact becomes complete
$\mathbf{J}$	Jacobian
η	peak density in a random process $[1/m^2]$
κ_1, κ_2	half of the positive maximum and minimum principle "curvatures" of the asperity of the "pressure surface", $p = -p_c(x,y)$ $[Pa/m^2]$
κ_1^*, κ_2^*	dimensionless principle curvatures of the local asperities, $\kappa_i^* = \kappa_i/\sqrt{m_4}, i = 1,2$
κ_m	mean (positive) curvature
$\mathcal{F}, \mathcal{F}^{-1}$	Fourier transform and inverse Fourier transform operators
ν	Poisson's ratio of equivalent rough surface
$\nu_i, i = 1,2$	Poisson's ratio of rough surface 1 and 2
σ	root mean square roughness of the surface
σ_s	root meant square of the asperity height
erfc()	complementary error function
erf()	error function
$\widetilde{A}_i$	tensile stress area, i.e., the area of tensile stress in $p_2(x,y)$ within each non-contact region
ξ, ζ	new coordinates, $\xi = x'$, $\zeta = \sqrt{\frac{\kappa_2}{\kappa_1}} y'$
$\xi_i, i = 1,\ldots,6$	random variables in the Nayak's random theory
a_n, b_n	semi-major and semi-minor axes of the elliptic non-contact region, $\overline{A}_n$
d	surface separation between mean levels of two nominally flat rough surface or between mean level of an effective rough surface and a rigid flat
d^*	dimensionless surface separation, $d^* = d/\sigma_s^h$ in original GW model and $d^* = d/\sqrt{m_0^h}$ in original Nayak–Bush and Greenwood's simplified model
e	eccentricity
$g(r)$	crack opening displacement of an axisymmetric mode-I crack
$g(x,y)$	gap distribution between the contact interfaces
h	height of the equivalent rough surface, $h = h_1 + h_2$
$h_i, i = 1,2$	height of rough surface 1 and 2, $E[h_i] = 0$
m	distance between mean level and mean asperity level
$m_i, i = 0,2,4$	spectral moments of an isotropic rough surface
p	normal contact pressure distribution acting on the boundary, $z = 0$, of a half-space
p_1	contact pressure distribution at complete contact (see Eq. (9))
p_2	contact pressure distribution acting only on the non-contact regions (see Eq. (10))
p_c	normal traction distribution at complete contact where $u_z(x,y) = h(x,y)$
q_x, q_y	tangential traction distributions in the x and y directions on the boundary, $z = 0$, of a half-space
r	polar coordinate, $r = \sqrt{\xi^2 + \zeta^2}$
s	semi-sum of the dimensionless principle curvatures of the local asperities: κ_1^* and κ_2^*, i.e., $s = -(\kappa_1^* + \kappa_2^*)$
u	dimensionless (negative) mean curvature, $u = -\kappa_m/\sqrt{m_4}$
$u_i, i = x,y,z$	surface displacement fields due to the given traction distributions on the boundary, $z = 0$, of a half-space
w	amplitude of the frequency vector $\mathbf{w}$
w_x, w_y	frequency components in the x and y directions
x', y'	local coordinates of each non-contact region centered about its centroid (see Fig. 6)
x, y, z	Cartesian coordinates
$\mathbf{E}$	complete elliptic integral of second kind
$\mathbf{w}$	frequency vector contains the frequencies in the x and y directions

Superscript

h	for the rough surface
p	for the "pressure surface"
$*$	dimensionless symbol, except for the effective material modulus, E^*

Abbreviation

GW	Greenwood and Williamson model
BGT	Bush, Gibson and Thomas model

contact area (likewise visualized as the "sea") when the average contact pressure is extremely high. The systematic study of nearly complete contact has received less attention compared to the early contact case even though it has many applications, such as the leakage of static seals, electrical contacts and tire/road interaction.

Johnson et al. (1985) derived the asymptotic solutions of the rough contact problem of an elastic half-space with slightly (bi-) sinusoidal waviness in contact with a rigid flat at nearly complete contact. They treated the gaps between the deformed waviness and the rigid flat as mode-I "cracks". Based on the concept of the *stress intensity factor* (SIF) in fracture mechanics, they obtained the approximate analytic solution to the relation between the average contact pressure and non-contact area within a complete period.