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Rapid simulation procedure for fretting wear on the basis of the method of dimensionality reduction

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ABSTRACT

We suggest a numerical procedure for rapid simulation of fretting wear in a contact of two bodies subjected to tangential oscillations with a small amplitude. The incremental wear in each point of contact area is calculated using the Reye–Archard–Khrushchov wear criterion. For applying this criterion, the distributions of pressure and relative displacements of bodies are required. These are calculated using the method of dimensionality reduction (MDR).

1. Introduction

Fretting wear occurs if two bodies are pressed against each other and are subsequently subjected to oscillations with small amplitude. Even if there is no gross slip in the contact, tangential slip occurs at the border of the contact area leading to wear and fatigue. Fretting wear was in the past an object of intensive experimental investigation and theoretical simulation for such applications as fretting of tubes in steam generators and heat exchangers (Ko, 1979; Fisher et al., 1995; Lee et al., 2009), joints in orthopedics (Collier, 1992), electrical connectors (Antler, 1985), and dovetail blade roots of gas turbines (Rajasekaran and Nowell, 2006; Ciavarella and Demelio, 2001) as well as many others. Most theoretical works were concerned with finite element (Ding et al., 2009; Mohd Tobia et al., 2009) or boundary element simulations (Lee et al., 2009). Thus in (Ding et al., 2007) a fretting wear modelling of complex geometries like spline coupling with finite element modeling was considered. Even while these simulations provided a complete picture of fretting wear, they still require too much computational time to be implemented as an interface in larger dynamic simulations. In a conventional finite element fretting simulation most of the time is wasted not for the calculation of wear itself but for the solution of the normal and tangential contact problems of progressively changing profile. That is why there are a lot of alternative approaches to a full finite element analysis. Examples of analytical and semi-analytical approaches were given in Nowell (2010) and Hills et al. (2009). In the present paper, we suggest to do this step using the method of dimensionality reduction (Heß, 2012; Popov, 2013; Popov and Heß, 2013; Popov, 2012). This drastically reduces the time of the whole simulation.

2. The method of dimensionality reduction

In this section we quickly recapitulate the main rules of the method of dimensionality reduction (Heß, 2012; Popov and Heß, 2013, 2014a). We consider a contact of a three-dimensional rotationally symmetric profile $z = I(r)$ and an elastic half-space. The profile is first transformed into a one-dimensional profile $g(x)$ according to the MDR-rule (Heß, 2012; Popov and Heß, 2013)

$$g(x) = |x| \int_0^{|x|} \frac{I'(r)dr}{\sqrt{x^2 - r^2}} \tag{1}$$

as illustrated in Fig. 1, where $I'(r)$ is a first derivative of $I(r)$.

The reverse transformation is given by the integral

$$I(r) = \frac{2}{\pi} \int_0^r \frac{g(x)}{\sqrt{r^2 - x^2}} dx \tag{2}$$

The profile (1) is pressed to a given indentation depth d into an elastic foundation consisting of independent springs with spacing

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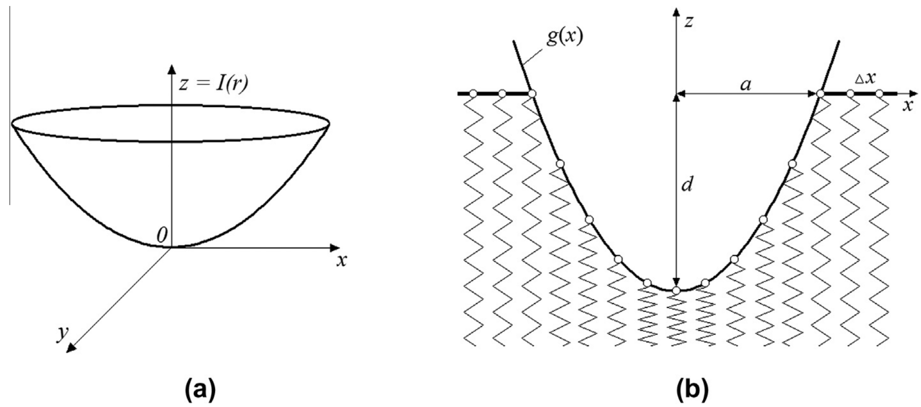

Fig. 1. The 3-dimensional body of revolution (a); and the corresponding one-dimensional MDR-transformed profile in a contact with the elastic foundation.

Δx (Fig. 1b) whose normal and tangential stiffness is given by (Popov and Heß, 2013)

$$\begin{aligned} k_z &= E^* \Delta x \\ k_x &= G^* \Delta x \end{aligned}, \tag{3}$$

where E^* is the effective elastic modulus

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \tag{4}$$

and G^* the effective shear modulus

$$\frac{1}{G^*} = \frac{(2-\nu_1)}{4G_1} + \frac{(2-\nu_2)}{4G_2}, \tag{5}$$

E_1 and E_2 are the Young's moduli, G_1 and G_2 the shear moduli of the indenter and the half -space, and ν_1 and ν_2 are their Poisson-ratios. Note that throughout this paper, we assume that the contacting materials satisfy the condition of "elastic similarity"

$$\frac{1-2\nu_1}{G_1} = \frac{1-2\nu_2}{G_2} \tag{6}$$

that guarantees the decoupling of the normal and tangential contact problems (Johnson, 1985). Note that the choice of the spatial step Δx is arbitrary as long as it is much smaller than all characteristic length scales of the problem; the solution does not depend on its choice.

The vertical displacement of an individual spring is given by

$$u_z(x) = d - g(x) \tag{7}$$

and the resulting normal force is given by

$$f_z(x) = E^* \Delta x (d - g(x)). \tag{8}$$

The linear force density is therefore

$$q_z(x) = \frac{f_z(x)}{\Delta x} = E^* u_z(x) = E^*(d - g(x)). \tag{9}$$

The contact radius a is determined by the condition

$$g(a) = d. \tag{10}$$

The total normal force is obtained by integration over all springs in contact:

$$F_N = 2E^* \int_0^a (d - g(x)) \mathrm{d}x. \tag{11}$$

According to the MDR rules, the distribution of normal pressure p in the initial three-dimensional problem can be calculated using the following integral transformation (Heß, 2012; Popov and Heß, 2013):

$$p(r) = -\frac{1}{\pi} \int_r^\infty \frac{q_z'(x)}{\sqrt{x^2 - r^2}} \mathrm{d}x = \frac{E^*}{\pi} \int_r^a \frac{g'(x)}{\sqrt{x^2 - r^2}} \mathrm{d}x. \tag{12}$$

Note that all above results obtained by the MDR, represent *exact* solutions of the corresponding three-dimensional problem. As was shown by Galin (1961), the transformation (1) maps the complete three-dimensional contact problem to a one-dimensional contact with an elastic foundation. All three-dimensional properties (as displacements, stresses and so on) can be obtained for the solution of the linear elastic foundation problem by appropriate integral transformations. This solution is *exact* and was used later in the well-known publication by Sneddon (1965). This solution can be generalized to all contact problems which can be reduced to the normal contact problem.

The complete proof for tangential contact can be found in the book (Popov and Heß, 2014b).

If the indenter is now moved in the tangential direction by $u_x^{(0)}$, the springs in contact will first stick to the indenter thus producing tangential force $f_x = k_x u_x^{(0)}$ until this force achieves the critical value μf_z, where μ is the coefficient of friction. After this, the tangential force remains constant and equal to μf_z while the springs begin to slide. The same is valid if the movement starts from an arbitrary stress state of a spring. It either follows the indenter, if the tangential force is smaller than the critical one or it slides, in which case the tangential force is equal to the critical value. Thus, for any incremental change of the tangential displacement the following equations are valid:

$$\begin{aligned} &\Delta u_x(x) = \Delta u_x^{(0)}, \quad \text{if} |k_x u_x(x)| < \mu f_z \\ &u_x(x) = \pm \tfrac{\mu f_z(x)}{k_x}, \quad \text{in the sliding state} \end{aligned}. \tag{13}$$

The sign in the last line of this equation depends on the direction of movement of the indenter. By following incremental changes in the indenter position, the absolute tangential displacement can be determined unambiguously at any location and any point in time. Therefore, the tangential force will also be determined:

$$f_x = k_x u_x(x) = G^* \Delta x \cdot u_x(x). \tag{14}$$

The tangential force density is equal to

$$q_x(x) = \frac{f_x}{\Delta x} = G^* u_x(x). \tag{15}$$

Distributions of tangential stresses $\tau(r)$ and displacements $u_x^{(3D)}(r)$ in the initial three-dimensional problem are defined by equations similar to (2) and (12), (Popov and Heß, 2014b):

$$u_x^{(3D)}(r) = \frac{2}{\pi} \int_0^r \frac{u_x(x) dx}{\sqrt{r^2 - x^2}}, \tag{16}$$

$$\tau(r) = -\frac{1}{\pi} \int_r^\infty \frac{q_x'(x) dx}{\sqrt{x^2 - r^2}} = -\frac{G^*}{\pi} \int_r^\infty \frac{u_x'(x) dx}{\sqrt{x^2 - r^2}}. \tag{17}$$