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Constitutive modeling of isotropic hyperelastic materials in an exponential framework using a self-contained approach

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ABSTRACT

In this paper, an exponential framework for strain energy density functions of elastomers and soft biological tissues is proposed. Based on this framework and using a self-contained approach that is different from a guesswork or combination viewpoint, a set strain energy density functions in terms of the first and second strain invariants is rebuilt. Among the constructed options for strain energy density, a new exponential and mathematically justified model is examined. This model benefits from the existence of second strain invariant, simplicity, stability of parameters, and the state of being accurate. This model can capture strain softening, strain hardening and is able to differentiate between various deformation-state dependent responses of elastomers and soft tissues undergoing finite deformation. The model has two material parameters and the mathematical formulation is simple to render the possibility of numerical implementations. In order to investigate the appropriateness of the proposed model in comparison to other hyperelastic models, several experimental data for incompressible isotropic materials (elastomers) such as VHB 4905 (polyacrylate rubber), two various silicone rubbers, synthetic rubber neoprene, two different natural rubbers, b186 rubber (a carbon black-filled rubber), Yeoh vulcanizate rubber, and finally porcine liver tissue (a very soft biological tissue) are examined. The results demonstrate that the proposed model provides an acceptable prediction of the behavior of elastomers and soft tissues under large deformation for different applied loading states.

1. Introduction

The most outstanding property of elastomers is their ability to undergo large deformation under relatively small stress. Over the last few decades, there have been considerable applications of them at real life scenarios as well as model developments based on different assumptions to predict the non-linear behavior of such materials. As example of these large stretches, rubber boot of an automotive CV joint that is simultaneously seeing large deformations and different loads and temperatures, large deformation analysis of dielectric membranes (Goulbourne et al., 2007), elastomeric polymer light-emitting devices and displays (Liang et al., 2013), stretchable organic solar cells (Lipomi et al., 2011), flexible and stretchable electrodes for dielectric elastomer actuators (Rosset and Shea, 2013), high strain rate response of rubber membrane (Albrecht and Ravi-Chandar, 2013), inflation of tubular elastomeric balloons (Mao et al., 2014) all rely on the remarkable large-stretch properties of elastomeric materials. Experimental evidences suggest that there exist three regions for engineering stress-stretch curves of rubbers: a large softening region with monotone increasing is followed by a moderate region and then often, it experiences an abrupt upturn under large strains. The computed results show rather abrupt changes in the slope of the curves at biological tissues than rubbers.

As Fig. 1 depicts, the behavior of elastomeric materials differs at different states of deformation. Another significant outcome of this simulation is that, a hyperelastic model could be handled well in uniaxial mode but might fail at another deformation. Considering this complexity in analysis for a real construction in the finite element method demands an accurate mechanical model. Accordingly, one often adopts a strategy that seeks to use the most accurate mechanical model as well as the simplest acceptable model that contributes a reasonable approximation of a deformation, usually leads to a weighted model with large number of parameters.

A considerable number of investigations have been conducted over the last decades on the elastomers and their large strain behaviors. Different constitutive models have been proposed to

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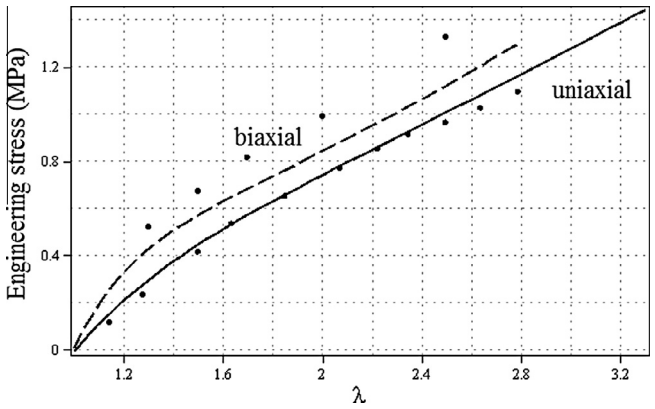

Fig. 1. Material behavior at different deformations, for example: uniaxial and biaxial deformations. The Arruda–Boyce model (1993) can follow experimental data accurately at uniaxial deformation but this model is not sufficient for another mode of deformation for gum rubber. Data is taken from James et al. (1975).

predict these characteristics which some require complex material investigation in order to derive material parameters. In the framework of the theory of isotropic hyperelasticity, strain energy functions may be divided in two overall categories: statistical models and phenomenological models.

Statistical mechanics models or chain models such as Arruda–Boyce 8-chain model (1993) appear to provide the most predictive model of the larger strain behavior under different states of deformation (Boyce and Arruda, 2000). Chain models take advantage of this useful property that they comprise minimum number of parameters, which are related to molecular quantities. It is nonetheless useful to mention that the four parameters Meissner and Matejka model (2003) is not related to molecular quantities, the four parameters Edwards and Vilgis model (1986) related to the molecular quantities and its form is so complicated, two number of five parameters of Miehe et al. model (2004) is not related to molecular features and Arruda–Boyce model (1993) fails at small deformation. Moreover, it is worth to mention two physically-based models in finite elasticity of elastomers which have been developed and applied to fit similar experimental data. The first model is by Drozdov and Gottlieb (2006) which coincides with the Ogden law by a special choice of adjustable parameters, and the second model is by Drozdov and Christiansen (2006) that is a constrained chain model with four parameters. Although chain models obtain many advantages for behavior modeling of the rubbers, it is not suitable for taking some important observed phenomena such as irreversible deformations and inelastic volumetric expansion (Gernay et al., 2013). As mentioned previously, chain models have a complicated structure, so that in some cases they are not amenable to provide a closed-form solution. Maybe this is the reason that Horgan and Saccomandi (2003, 1999) and others have a tendency to use more simple strain energy functions for analytical closed-form solution to boundary-value problems instead of complicated chain models. Moreover, chain models, due to complicated forms, are not a desirable for numerical solutions.

The phenomenological models are supported by a mathematical relationship and treat the problem from viewpoint of continuum mechanics. In this viewpoint, strain energy function may be modeled as invariant-base and principal stretch-base. For example, Mooney (1940) published an invariant-base phenomenological model in terms of principal invariants of the right Cauchy–Green strain tensor, I_1 and I_2. Later, Treloar (1943a,b) proposed a so-called Neo-Hookean material model in terms of I_1 with only one material parameter; although, these two models are restricted to the small deformations. Rivlin (1948) introduced a generalized model, also called polynomial hyperelastic model in terms of strain invariants. Following this structure, several investigators attempted to consider strain invariants in their models in the framework of polynomial hyperelastic model. As examples of these models Yeoh (1993), Isihara et al. (1951), Biderman (1958), James et al. (1975), Lopez-Pamies (2010), Attard and Hunt (2004) and Hartmann and Neff (2003) altogether are polynomial forms of strain energy and encompass high order strain invariants. Tschoegl (1971) emphasized that keeping of higher order terms in the generalized Mooney–Rivlin polynomial function leads to a better correlation with test data for both filled and unfilled rubbers. Although high-order polynomial models, in terms of strain invariants, fit the almost any hyperelastic experimental curve, but they can also introduce difficulties during numerical solution (Meunier et al., 2007, 2008). Hence, researchers prefer to use a complete yet as simple as possible constitutive model for numerical and analytical solutions. The two-parameter Gent model (1996) is a first invariant-based and has some attractive features, so that Boyce (1996) compared this model with the 8-chain model and realized that they are almost identical to structure and qualities of test results. Also, Pucci-Saccomandi model (2002) and Yeoh-Fleming model (1997) combine the concept of Gent. The other strain energy functions include Gent and Thomas (1958), Hart-Smith (1966), Carroll (2011), Khajehsaeid et al. (2013) and Nunes (2011) for modeling the nonlinear elastic response of elastomers. Another brilliant stretch-base strain energy function has been proposed by Ogden (1972). It delivers a good agreement with Treloar's experimental data for extension of unfilled natural rubber.

There exists several comprehensive comparison and excellent review of the development of phenomenological and statistical mechanics treatment of rubber elasticity at different literatures, e.g., Elıas-Zúñiga and Beatty (2002), Boyce and Arruda (2000), Marckmann and Verron (2006), Steinmann et al. (2012), Martins et al. (2006) and Vahapoglu and Karadeniz (2006). As a result of these investigations, a large number of well-known models are not reliable on the entire ranges of strain and different modes of deformation, simultaneously. Furthermore, they might be failed to be well matched with different materials.

In this work, we propose a new framework for strain energy density function of elastomers and soft tissues (Section 2). In Section 2, based on this framework we construct a set strain energy density functions in terms of the first and second strain invariants. Among this set, we select a simple exponential mathematical model with two material parameters for the behavior modeling of a wide range of rubber-like materials and biological tissues at different states of deformation. In Section 3, the calculation method of the material parameters is presented. In Section 4, we fit the proposed model to different test data to demonstrate the model's performance in describing rubber-like and biological materials. As a result of these comparisons, the model is able to capture mechanical behavior of such materials. In Section 5, the significance of different terms in the proposed model is discussed, comprehensively.

2. Model development

The general motion of a continuum is described by $\mathbf{x} = \chi(\mathbf{X}, t)$, where $\mathbf{X}$ and $\mathbf{x}$ denote the position vectors of material particle in its reference configuration and current configuration at time t, respectively. The deformation gradient is shown by $\mathbf{F} = \partial\mathbf{x}/\partial\mathbf{X}$. Since $\det(\boldsymbol{F}) > 0$, the polar decomposition theorem states that F is uniquely decomposed as

$$\mathbf{F} = \mathbf{RU} = \mathbf{VR} \tag{1}$$

where $\mathbf{U}$ and $\mathbf{V}$ are the right and left stretch tensors, respectively. $\mathbf{U}$ and $\mathbf{V}$ are positive definite symmetric tensors and $\mathbf{R}$ is a proper