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Cohesive zone finite element analysis of crack initiation from a butt joint's interface corner

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ABSTRACT

Cohesive zone (CZ) fracture analysis techniques are used to predict the initiation of crack growth from the interface corner of an adhesively bonded butt joint. In this plane strain analysis, a thin linear elastic adhesive layer is sandwiched between rigid adherends. There is no preexisting crack in the problem analyzed, and the focus is on how the shape of the traction–separation (T–U) relationship affects the predicted joint strength. Unlike the case of a preexisting interfacial crack, the calculated results clearly indicate that the predicted joint strength depends on the shape of the T–U relationship. Most of the calculations used a rectangular T–U relationship whose shape (aspect ratio) is defined by two parameters: the interfacial strength σ^* and the work of separation/unit area Γ. The principal finding of this study is that for a specified adhesive layer thickness, there is any number of σ^*, Γ combinations that generate the same predicted joint strength. Each combination corresponds to a different CZ length. An approximate CZ-like elasticity solution was developed to show how such combinations arise and their connection with the CZ length.

1. Introduction

The cohesive zone (CZ) fracture modeling technique is now commonly used to predict failure of bodies containing an initial crack (Tvergaard and Hutchinson, 1993; Xu and Needleman, 1994) and adhesively bonded joints (Yang and Thouless, 2001; Kafkalidis and Thouless, 2002; Blackman et al., 2003; Liljedahl et al., 2006; Banea and da Silva, 2009; Gustafson and Waas, 2009). When used in an analysis where all bulk materials are linear elastic, one recovers linear elastic fracture mechanics predictions provided that the CZ is sufficiently small when compared to the crack length (i.e., similar to a small-scale yielding requirement). Even though the shape of the traction–separation (T–U) relationship used in a CZ fracture analysis affects the length of the CZ, this change in length has negligible effect if the CZ is sufficiently small. In such cases the solution only depends on the area under the T–U relationship, which equals the work of separation/unit area of crack advance (i.e., fracture toughness).

Although most frequently applied to cracked bodies, a CZ failure analysis has also been applied with some success to problems where there is no preexisting crack, but where failure initiates from sharp discontinuities such as generated by corners or sharp notches. For example, CZ modeling techniques have been applied to V-notched PMMA samples with various notch angles, depths, and sizes (Gomez and Elices, 2003). In this study, the specimens were loaded either in tension or bending. Predicted strengths were generally in good agreement with the experimental results. These authors indicated that they found that the rectangular T–U relationship was the best shape to reproduce all experimental results. The reason why this is true was not discussed. In another study, a CZ analysis was used to predict the initiation of crack growth from the bimaterial corner of an aluminum/epoxy specimen (Mohammed and Liechti, 2000). In this work, a CZ model was calibrated using experimental data for an interfacial crack and then used to successfully predict the strength of specimens with varying corner angles. The present work examines the use of a CZ fracture analysis to predict the strength of a sharp-edged, adhesively bonded butt joint. This type of joint is commonly used to evaluate adhesives and is also a relatively simple geometry to analyze.

2. Failure analysis of an adhesively bonded butt joint based on a critical value of the interface corner stress intensity factor

In previous work, a method analogous to traditional fracture mechanics was found to accurately predict the strength of sharp-edged, adhesively bonded butt joints (Reedy, 1990; Reedy and Guess, 1993, 1997, 1999). This technique uses the stress intensity factor associated with the interface corner (IC) discontinuity

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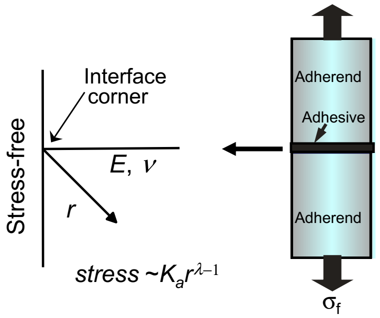

Fig. 1. Adhesively bonded, sharp-edged cylindrical butt joint with the associated idealized plane strain asymptotic problem of an elastic quarter-plane bonded to a rigid quarter-plane.

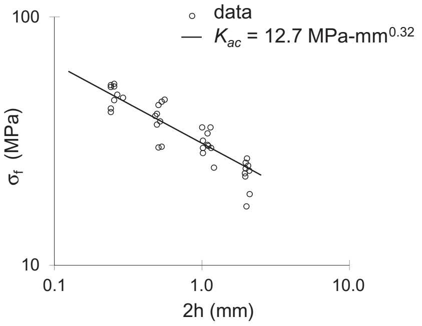

Fig. 2. Comparison of measured butt joint tensile strength vs. bond thickness data with prediction based on an interface corner toughness of 12.7 MPa-mm$^{0.32}$.

(Fig. 1). The stress state in the region of the IC varies as $K_a r^{\lambda-1}$ where r is the distance from the IC and $\lambda - 1$ is the order of the power-law singularity (which is weaker than that found at a crack tip). The value of the IC stress intensity factor K_a determines the magnitude of the stress state in the region of the interface corner. It depends on loading, geometry, and layer elastic properties. For a thin adhesive layer sandwiched between rigid adherends

$$K_a = \frac{\nu}{1-\nu}\sigma_n h^{1-\lambda} A(\nu) \tag{1}$$

where $A(\nu)$ is a function of Poisson's ratio ν, $2h$ is the thickness of the adhesive layer, and σ_n is the nominal applied tensile stress (applied load/cross sectional area). The strength of the singularity also depends on ν. When $\nu = 0.35$, $A(\nu) = 0.958$, and $1 - \lambda = 0.320$.

The IC failure theory for adhesively bonded butt joints postulates that fracture initiates once the surrounding stress field reaches a critical state

$$K_a \text{ (loading)} = K_{ac} \text{ (material property)} \tag{2}$$

where the critical value of the IC stress intensity factor K_{ac} is a measured material property and is referred to as the IC toughness. This approach requires that the asymptotic stress state characterized by K_a must dominate a region about the interface corner that is significantly larger than the fracture process zone, intrinsic flaw size, and the plastic yield zone (i.e., requirements similar to small scale yielding in linear elastic fracture mechanics). This technique was found to accurately predict the observed variation in joint strength with bond thickness. If K_a is equal to K_{ac}, then Eq. (1) requires $\sigma_f h^{1-\lambda}$ to remain constant, where σ_f is the nominal butt joint tensile strength (failure load/cross sectional area). The measured tensile strength of butt joints formed by bonding 28.6-mm diameter, stainless steel adherends together with an epoxy adhesive was found to follow the predicted power-law relationship between joint strength and bond thickness as shown in Fig. 2. A detailed description of the butt joint tests that measured the strength data plotted in Fig. 2 is documented elsewhere (Reedy and Guess, 1993). Note that in these tests the plastic yield zone emanating from the interface corner is estimated to be less than 2% of the bond thickness, and consequently, the small scale yielding idealization applies. Similar levels of agreement with butt joint strength data have been observed for other butt joint tests (Reedy and Guess, 1997, 1999). One of the questions being addressed in this study is whether a CZ failure analysis could provide an alternate approach for predicting the strength of adhesively bonded butt joints.

3. CZ Fracture analysis of an adhesively bonded butt joint

In a CZ model, interfacial separation is defined in terms of an effective interfacial traction vs. separation relationship (Fig. 3). Key parameters defining this T–U relationship are the interfacial strength σ^* and the work of separation/unit area Γ. A CZ separation model is computationally attractive for simulating interfacial failure since crack growth is a natural outcome of the solution, and moreover it leads to mesh-independent results since a length scale is embedded within the model (provided that the mesh is fine enough to resolve the CZ — the region of interfacial softening behind the crack tip). The particular CZ formulation used in this study is similar to that used by Tvergaard and Hutchinson (1993). The effective separation χ is defined as

$$\chi = \sqrt{\left(\frac{\delta_n}{\delta_n^c}\right)^2 + \left(\frac{\delta_t}{\delta_t^c}\right)^2} \tag{3}$$

where δ_n and δ_t are the normal and tangential displacement jump across the interface while δ_n^c and δ_t^c are the respective critical values. Since there is no compelling reason to assume otherwise, $\delta_n^c = \delta_t^c = \delta_c$ is assumed. The normal and tangential interfacial tractions (T_n and T_t, respectively) are defined via the potential

$$\phi(\delta_n, \delta_t) = \delta_c \int_0^{\chi} \sigma(\chi')d\chi' \tag{4}$$

with

$$T_n = \frac{\partial\phi}{\partial\delta_n} = \frac{\sigma(\chi)}{\chi}\frac{\delta_n}{\delta_c} \quad \text{and} \quad T_t = \frac{\partial\phi}{\partial\delta_t} = \frac{\sigma(\chi)}{\chi}\frac{\delta_t}{\delta_c} \tag{5}$$

Normal interpenetration is penalized by applying a prescribed multiple of the initial loading stiffness. Unless indicated otherwise, a trapezoidal T–U relationship was used, where χ_1 and χ_2 define its shape. The trapezoidal T–U relationship was chosen for its simplicity and a relationship with steep loading and unloading slopes was used ($\chi_1 = 0.01$ and $\chi_2 = 0.99$). Consequently, the shape of the T–U relationship is essentially rectangular and will be referred to as such. The work of separation per unit area of interface (i.e., intrinsic interfacial toughness) is path independent and equals the value of the potential ϕ evaluated at $\chi = 1$ (Eq. (4)). For the assumed

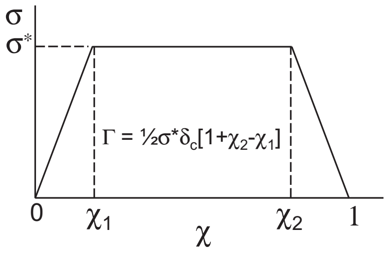

Fig. 3. Effective T–U relationship used in conjunction with the CZ model (in this study $\chi_1 = 0.01$ and $\chi_2 = 0.99$, and the T–U is approximately rectangular).