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Energy release rates in rubber during dynamic crack propagation

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ABSTRACT

The theoretical understanding of the fracture mechanics of rubber is not as well developed as for other engineering materials, such as metals. The present study is intended to further the understanding of the dissipative processes that take place in rubber in the vicinity of a propagating crack tip. This dissipation contributes significantly to the total fracture toughness of the rubber and is therefore of great interest from a fracture mechanics point of view. To study this, a computational framework for analysing high-speed crack growth in a biaxially stretched rubber under plane stress is therefore formulated. The main purpose is to investigate the energy release rates required for crack propagation under different modes of biaxial stretching. The results show, that inertia comes into play when the crack speed exceeds about 50 m/s. The total work of fracture by far exceeds the surface energy consumed at the very crack tip, and the difference must be attributed to dissipative damage processes in the vicinity of the crack tip. The size of this damage/dissipation zone is expected to be a few millimetres.

1. Introduction

Fracture mechanics of metals has been studied extensively both experimentally and theoretically over the years. When it comes to rubber, several experimental investigations have been made, e.g. Rivlin and Thomas (1953), Thomas (1955), Greensmith and Thomas (1955), Greensmith (1956), Kolsky (1969), Gent and Kim (1978), Gent and Marteny (1982a), Dragoni and Medri (1988), Chung et al. (1991), Deegan et al. (2002), Petersan et al. (2004), Zhang et al. (2009), Niemczura and Ravi-Chandar (2011) and Chen et al. (2011). Several of the early studies aimed at establishing a critical tearing energy for rubber. Other studies have tried to determine the crack speed as a function of the applied (macroscopic) load state. Additional information has been provided in other studies that have performed microscopic investigations of the fracture surfaces of rubber (Bascom, 1977; Fukahori and Andrews, 1978; Bhowmick et al., 1980; Setua and De, 1983; Gent and Pulford, 1984; Goldberg et al., 1988; Pandey and Mathur, 2003). However, the theoretical understanding of fracture in rubber is less developed.

Rubbers may be characterised as viscoelastic, which means that viscoelastic dissipation contributes to the total fracture toughness of the material. Hence, there are at least two different sources for energy consumption at an advancing crack tip in rubber. The first source is associated with the innermost region at the crack tip, where cavities form, polymer chains are pulled out, and polymer chain bonds are broken. This process is mainly governed by the basic molecular structure and strength of the material (Thomas, 1994). The second source is the viscoelastic dissipation in the polymer in front of the crack tip (Persson et al., 2005). Possibly some amount of damage may also be involved. On the basis of these observations, it is to be expected that the tearing energy of rubber will depend strongly on both temperature and crack velocity.

It has also been noted that transverse stretching (i.e. stretching in the direction of crack extension and propagation) tends to decrease the energy release rate for propagating cracks (Gent and Kim, 1978; Gent and Marteny, 1982a). In fact, rubber sheets that are highly stretched in the transverse direction can be split apart quite easily. This indicates that the originally isotropic material becomes fibrous in character, i.e. much weaker for a tear running in the direction of extension than for one running at right angles to it.

There are a few theoretical studies that examine the contribution of viscoelasticity to the fracture toughness of polymers, e.g. Carbone and Persson (2005), Marder (2005, 2006), Persson and Brener (2005), Wang and Chen (2005), Tang et al. (2008, 2009), Kroon (2011), Elmukashfi and Kroon (2012) and Elmukashfi and Kroon (in press). However, several of these studies adopt a theory valid for infinitesimal strains, and the validity of those results for rubber-like solids is therefore questionable. In a previous study by the present author (Kroon, 2011), dynamic crack propagation under steady-state conditions in rubber was examined. Plane deformation was assumed and a Kelvin-type of material was

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adopted. It was demonstrated that viscoelastic dissipation may explain the large discrepancies observed between the surface energy consumed at the very crack tip and the macroscopic fracture toughness obtained in experiments. Other theoretical studies address different aspects of crack growth in viscoelastic solids, mostly under the assumption of infinitesimal strains (Knauss, 1969; Lindley, 1973; Schapery, 1975, 1989; de Gennes, 1996; Rahulkumar et al., 2000).

In the present work, a computational framework is proposed for analysing high-speed crack propagation in biaxially stretched rubber sheets under conditions of steady-state and plane stress. The main purpose of the study is to shed some more light on the contributions from local surface energy at the crack tip and bulk viscoelasticity and possible damage processes to the total work of fracture required to propagate a crack. The problem to be analysed is formulated in Section 2 and the numerical implementation is outlined in Section 3. Numerical results are presented in Section 4 and discussed in Section 5.

2. Problem formulation

2.1. Geometry and boundary conditions

Consider a crack that propagates through a rubber solid at a constant speed and under steady-state conditions. The crack is illustrated in Fig. 1(a). At some point the crack passes through the control volume, indicated by the dotted box in Fig. 1(a). This rectangular control volume is analysed, and its dimensions and the applied boundary conditions are indicated in Fig. 1(b). Due to symmetry, only the upper half of the control volume is modelled. The coordinate system X_1–X_2–X_3 pertains to the reference configuration and is located at the propagating crack tip. Position vectors $\mathbf{x} = x_i\mathbf{e}_i$ and $\mathbf{X} = X_i\mathbf{e}_i$ are associated with the deformed and reference configurations, respectively, where $\mathbf{e}_i$ is a set of orthogonal basis vectors. The displacement vector is defined as $\mathbf{u} = \mathbf{x} - \mathbf{X}$, and the traction vector $\mathbf{T}$ is defined as force per unit undeformed area. Boundary conditions are applied according to

$$X_1 = \pm B_0 \;: u_1 = \pm\Delta_1, \quad T_2 = 0, \tag{1}$$

$$X_1 > 0, \quad X_2 = 0 \;: u_2 = 0, \quad T_1 = 0, \tag{2}$$

$$X_1 < 0, \quad X_2 = 0 \;: T_1 = T_2 = 0, \tag{3}$$

$$X_2 = H_0 \;: u_2 = \Delta_2, \quad T_1 = 0, \tag{4}$$

i.e. loading is imposed in terms of the prescribed displacements Δ_1 and Δ_2. In addition, plane stress is assumed, implying that $T_3 = 0 \; \forall \; \mathbf{X}$.

The applied boundary displacements correspond to the global stretches

$$\Lambda_1 = \frac{B_0 + \Delta_1}{B_0}, \quad \Lambda_2 = \frac{H_0 + \Delta_2}{H_0}. \tag{5}$$

2.2. Equations of motion

We assume that steady-state prevails, and the fundamental assumption is made that the time derivative of all field variables can be computed as

$$\frac{\mathrm{d}(\bullet)}{\mathrm{d}t} = -V_c\frac{\partial(\bullet)}{\partial X_1}, \tag{6}$$

where V_c is the Lagrangian speed of the crack in the reference configuration. The true crack speed v_c relates to the Lagrangian speed as

$$v_c = V_c\Lambda_1. \tag{7}$$

The equations of motion may be expressed as

$$\frac{\partial P_{ij}}{\partial X_j} = \rho_0\frac{\mathrm{d}^2 u_i}{\mathrm{d}t^2} = \rho_0 V_c^2\frac{\partial^2 u_i}{\partial X_1^2}, \tag{8}$$

where P_{ij} are the components of the first Piola–Kirchhoff stress tensor, and ρ_0 is the density of the material in the undeformed state. Body forces are ignored. Multiplication with a virtual displacement field δu_i and integration over the control volume domain Ω_0 gives

$$\int_{\Omega_0}\left(\frac{\partial P_{ij}}{\partial X_j} - \rho_0 V_c^2\frac{\partial^2 u_i}{\partial X_1^2}\right)\delta u_i \mathrm{d}\Omega_0 = 0. \tag{9}$$

By use of the chain rule and Gauss' theorem, Eq. (9) may be recast into

$$\int_{\Omega_0}\left(P_{ij} - \rho_0 V_c^2\frac{\partial u_i}{\partial X_1}\delta_{1j}\right)\frac{\partial \delta u_i}{\partial X_j}\mathrm{d}\Omega_0 + \int_{S_0}\left(\rho_0 V_c^2\frac{\partial u_i}{\partial X_1}N_1 - T_i\right)\delta u_i \mathrm{d}S_0 = 0, \tag{10}$$

where δ_{ij} is the Kronecker delta, S_0 is the boundary surface of the control volume in the reference configuration, N_i is the outward normal to S_0, and $T_i = P_{ij}N_j$ is the traction vector acting on S_0. We also note that

$$P_{ij}\frac{\partial \delta u_i}{\partial X_j} = P_{ij}\delta F_{ij} = S_{ij}\delta E_{ij}, \tag{11}$$

where $F_{ij} = \partial x_i/\partial X_j$, S_{ij}, and E_{ij} are the components of the deformation gradient tensor, the second Piola–Kirchhoff stress tensor, and the Green strain tensor, respectively. Application of the boundary

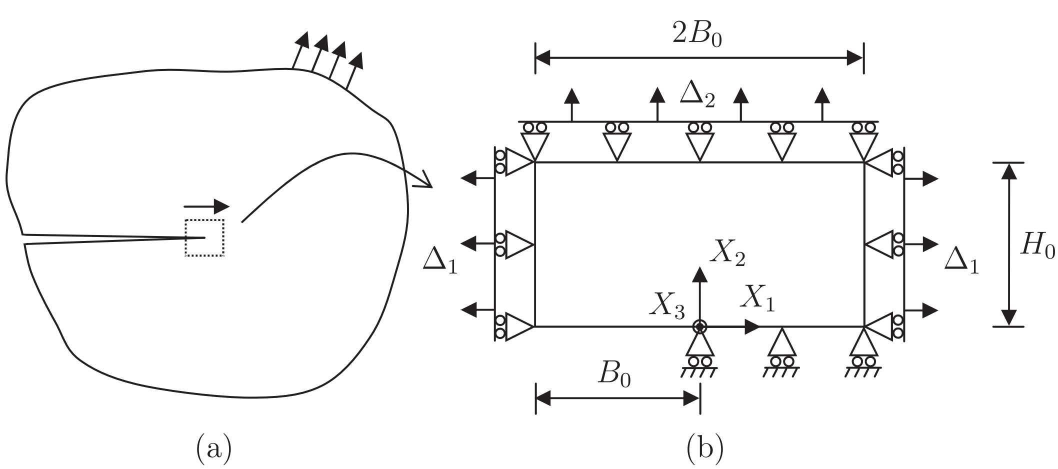

Fig. 1. (a) Crack propagating through a rubber solid. (b) Geometry and boundary conditions of the analysed control volume.