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Contact problems involving beams

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ABSTRACT

Elastic contact problems involving Euler–Bernoulli beams or Kirchhoff plates generally involve concentrated contact forces. Linear elasticity (e.g. finite element) solutions of the same problems show that finite contact regions are actually developed, but these regions have dimensions that are typically of the order of the beam thickness. Thus if beam theory is appropriate for a given structural problem, the local elasticity fields can be explored by asymptotic methods and will have fairly general (problem independent) characteristics. Here we show that the extent of the contact region is a fixed ratio of the beam thickness which is independent of the concentrated load predicted by the beam theory, and that the distribution of contact pressure in this region has a universal form, which is well approximated by a simple algebraic expression.

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1. Introduction

If classical Euler–Bernoulli beam theory is used to describe elastic components in frictionless contact, the solutions generally predict concentrated contact forces — i.e. that the extent of the contact area is restricted to one or more isolated points. A simple example is illustrated in Fig. 1, where a beam of length L is simply supported at its ends, and a rigid cylinder of radius R is pressed against it at the mid point by a force P.

In this situation, contact will occur only at the mid-point as long as the radius of curvature of the deformed beam is greater than R and this condition is satisfied if

$$P < P_0 \equiv \frac{4EI}{LR}, \tag{1}$$

where EI is the flexural rigidity of the beam.

For $P > P_0$, a finite strip of contact is developed — i.e. the beam conforms to the shape of the cylinder over a line segment of length a, but non-zero tractions are limited to a pair of concentrated forces $P/2$ at the two edges of this segment (Johnson, 1985), as shown in Fig. 2. The beam is then essentially loaded in 'four-point bending' and the bending moment in the contact segment is $P(L-a)/4$, corresponding to a radius of curvature $4EI/P(L-a)$. Equating this to the radius of the cylinder and solving for the length a, we obtain

$$a = L - \frac{4EI}{PR} = L\left(1 - \frac{P_0}{P}\right). \tag{2}$$

1.1. Higher order beam theories

Clearly the continuum solution of this problem will not involve concentrated forces, with the corresponding implication of locally unbounded stresses and strains. Instead, we anticipate the development of small but finite regions of contact with correspondingly large local contact stresses, whose value may be of importance for design purposes.

Some degree of regularization in the beam solution can be achieved by using higher order theories, such as Timoshenko beam theory (Chen, 2011), or by including the effect of transverse normal strain (Naghdi and Rubin, 1989; Gasmi et al., 2012). However, the resulting theories are considerably more complex to apply, and the contact pressure distributions still exhibit significant deviations from the 'exact' solution, particularly at the edges of the predicted contact region, where asymptotic arguments require that the contact pressure should go to zero with a square-root bounded form (Johnson, 1985).

1.2. Analytical solutions

The problem of Fig. 1 was solved exactly in the context of elasticity theory by Keer and Miller (1983), by expressing the elastic fields in the beam as Fourier transforms with respect to

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the horizontal variable. The lower surface of the beam is traction-free, and because the contact conditions are frictionless, the shear traction on the upper surface is also zero everywhere. Thus, three of the four boundary conditions are ‘global’ and can be satisfied by elementary relations between the transform variables. The remaining (normal) conditions on the upper surface then lead to a pair of dual integral equations and these can be reduced to a single Fredholm equation that must be solved numerically.

1.3. Finite element solutions

The Fourier transform technique has been applied to a range of beam-like contact problems (Schonberg et al., 1987; Keer and Schonberg, 1986; Keer and Silva, 1972) of which it clearly represents the definitive solution. However, its use demands a significant familiarity with dual integral equations and the final calculation still involves a numerical solution. A more straightforward alternative is of course to solve the complete structural contact problem using a two or three-dimensional finite element model, in which the contact tractions can be approximated to any desired degree of accuracy by suitable mesh refinement. However, this approach has its own problems, notably because (i) the resulting contact areas are very small and hence require very fine local meshing, but (ii) in many cases (for example, for the problem of Fig. 1 with $P > P_0$), the exact location of the contact region is not known *a priori*, so this fine mesh may need to be extended over a substantial region of the body.

1.4. Asymptotic arguments

The fact that the local contact stress fields will be restricted to a region that is small compared with the other dimensions of the problem opens up the possibility of using asymptotic methods. These methods have been used to great effect in deducing the character of the local frictional slip zones and stress fields in fretting fatigue applications, from parameters defined in the simpler, fully adhered solution (Churchman and Hills, 2006; Flicek et al., 2013).

In the problem of Fig. 1, if we choose a coordinate system centered on one of the two contact regions implied by the geometry of Fig. 2, and magnify the scale sufficiently for the resulting finite contact area to occupy most of the field of interest, then the magnification will usually be sufficient for the ends of the beam and the other region of contact to appear a large distance away. St. Venant’s principle then suggests that the effects of these distant loads can influence the local contact region only through the local values of bending moment and shear force, and hence it should be possible to characterize the local contact fields in terms of a quite limited number of parameters. In other words, we should be able to develop a few fairly general continuum contact solutions that can be ‘patched in’ to beam contact problems, enabling the maximum contact pressure and other parameters of interest to be predicted without necessitating a full continuum solution of the each individual problem. This is the objective of the present paper.

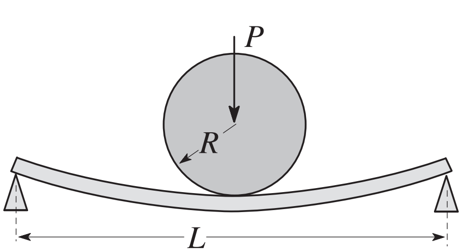

Fig. 1. A cylinder pressed against a beam.

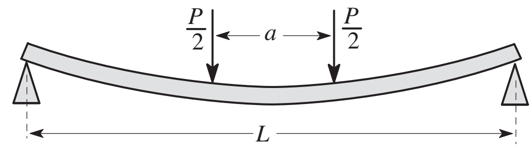

Fig. 2. Contact tractions for $P > P_0$.

2. Hertzian approximation

We consider the two-dimensional plane strain problem in which the beam of Figs. 1 and 2 is of depth h and the force P is to be interpreted as force per unit length (into the paper). We then have

$$EI = \frac{E^* h^3}{12} \quad \text{and} \quad P_0 = \frac{E^* h^3}{3LR}, \tag{3}$$

where

$$E^* = \frac{E}{(1 - \nu^2)} \tag{4}$$

is the plane strain modulus, and E, ν are Young’s modulus and Poisson’s ratio respectively.

If $P \ll P_0$, it seems reasonable to expect that the local contact behavior in Fig. 1 will be well approximated by the Hertzian equations. In particular, that the contact pressure distribution will be given by (Johnson, 1985)

$$p(x) = \frac{2P\sqrt{b^2 - x^2}}{\pi b^2} \tag{5}$$

and the contact semi-width b will be

$$b = 2\sqrt{\frac{PR}{\pi E^*}}. \tag{6}$$

We might hope to obtain a better approximation to the local fields by recognizing that in the beam solution, the contact surface is concave with radius

$$R_b = \frac{4EI}{PL}. \tag{7}$$

This value is determined by the bending moment in the beam, which is only very slightly affected by the exact contact pressure distribution, so we can reasonably treat it as a pre-existing radius and calculate $p(x)$ and b by replacing R by the composite radius R^* where

$$\frac{1}{R^*} = \frac{1}{R} - \frac{1}{R_b} = \frac{1}{R}\left(1 - \frac{P}{P_0}\right). \tag{8}$$

We obtain

$$b = 2\sqrt{\frac{h^3}{3\pi L}\left(\frac{\tilde{P}}{1 - \tilde{P}}\right)} \quad \text{where} \quad \tilde{P} = \frac{P}{P_0}, \tag{9}$$

after which $p(x)$ is given by (5). In particular, the maximum contact pressure is

$$p(0) = P_0\sqrt{\frac{3L\tilde{P}(1 - \tilde{P})}{\pi h^3}}. \tag{10}$$

3. Finite element solution

To evaluate the range in which these approximations are appropriate, we constructed a finite element model of the problem. The

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