International Journal of Solids and Structures 51 (2014) 3807–3822

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Influence of the fiber geometry on the macroscopic elastic and thermal properties

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ARTICLE INFO

Article history:
Received 24 September 2013
Received in revised form 4 May 2014
Available online 30 June 2014

Keywords:
Glass fiber reinforced polymer
FFT computation
Stochastic modeling
Linear elasticity
Thermal conductivity
Representative volume element

ABSTRACT

This work focuses on the linear elastic and thermal properties of real and virtual, computer-generated fibrous composites. A stochastic microstructure model is used to generate densely-assembled 3D systems of curved, non overlapping fibers with specific orientation distributions. This model is first optimized to approach the characteristics of a real fiber glass polymer by fitting geometrical and statistical parameters, such as fiber orientation, radius, length, and curvature. Second, random realizations of the stochastic models that depart from the characteristics of the fiber glass polymer are generated. The latter, which range from isotropic to transversely isotropic and to orthotropic materials, represent plausible virtual fibrous materials. Full-field numerical computations, undertaken by means of the Fourier-based (FFT) method, are used to estimate the local and effective mechanical and thermal responses of the fibrous composites. The anisotropy of the macroscopic responses as well as the size of the corresponding representative volume element (RVE) are examined numerically. It is found that the variance of the properties on a volume V scales as a powerlaw $\sim 1/V^{\alpha}$ where $\alpha < 1$, an effect of long-range correlations in the microstructure. Finally, the overall behavior of the fiber composites are computed for varying fiber curvature and orientation distributions, and compared with available analytical bounds. We find that the fiber arrangement strongly influences the elastic and thermal responses, less so for the fiber curvature.

1. Introduction

The use of fibrous composites has recently increased in vast areas of material engineering, such as in aeronautics and automotive industry. Their physical properties largely depend upon the materials microstructure and fibers arrangement which 2D models can not take into account; accordingly, a detailed analysis of these materials is required to estimate and understand their macroscopic behavior. The final goal is to optimize fibrous materials by means of "virtual material design". New fibrous materials are virtually created as realizations of a stochastic model and evaluated with physical simulations. This allows for material optimization for a specific use, without constructing expensive prototypes or performing mechanical experiments.

In order to design a practically fabricable material, a stochastic model is designed and adapted to an existing material and then slightly modified. The virtual reconstruction of the existing material requires a precise knowledge of the geometry of its microstructure. We propose and apply a local analysis of fiber orientation and radius as well as a single fiber tracking approach to characterize in details the fiber system. In this work, the theory used to simulate and interpret the elastic and thermal properties of fiber-reinforced materials is presented, and then applied to a glass-fiber reinforced polymer.

This paper is structured in three parts. The first part on image analysis and stochastic modeling is used to generate realizations of virtual materials of fiber systems. The second part is devoted to the local and effective constitutive laws of the material, in mechanics and conductivity, as well as the numerical method used to solve them. Finally, the method is applied to a sample of glass fiber reinforced composite.

2. Microstructure characterization and stochastic models for fibrous composites

In this section, the fibrous microstructure, its virtual reconstruction, that serves as a basis for the physical simulations, and

Abbreviations: FFT, fast Fourier transform; RVE, representative volume element; GRP, glass-fiber reinforced polymer; HS, Hashin–Shtrikman; MPa, mega pascal.

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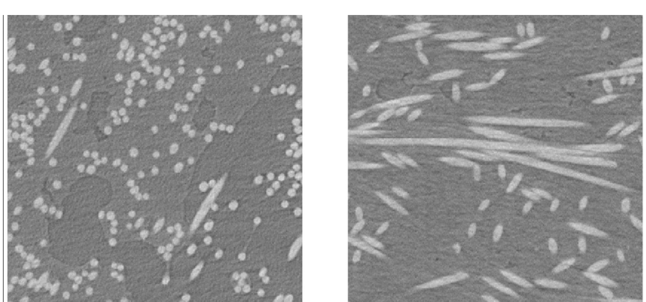
http://dx.doi.org/10.1016/j.ijsolstr.2014.05.013
0020-7683/© 2014 Published by Elsevier Ltd.

the stochastic fiber models are presented. We first introduce the material, glass fiber reinforced polymer, which is the subject of our application. A fiber tracking approach enables the complete quantification of the fiber system with amount, length, radius and curvature. Realizations of two stochastic models are considered: the standard Boolean system of (straight) cylinders and a complex fiber packing approach, more representative of the measured characteristics of the real microstructure.

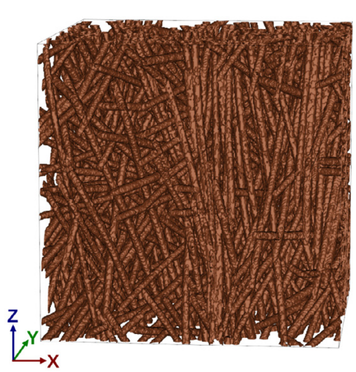
2.1. Glass-fiber reinforced polymer

Glass-fiber reinforced polymer (GRP) consists of a polymer matrix reinforced with thin glass fibers. Strong mechanical properties, i.e. high strength, are achieved when the glass fibers are free of defects. Full glass material without defects would have comparable strength, however, in contrast to glass fibers, it is practically impossible to build defect-free full glass materials. The main properties of GRP are light weight, extreme strength, and robustness. In comparison to carbon fiber reinforced polymers, the GRP has lower strength and is less stiff. Still, the GRP is typically far less brittle, and the raw materials are less expensive. A GRP is stiff and strong in tension and compression along the mean fiber alignment. In other directions, i.e. orthogonal to their principal axis, the glass fiber is neither stiff nor strong with respect to shear. Therefore, the orientation distribution of the fibers plays a centered role for its physical behavior.

Common uses of GRP include boats, automobiles, baths, hot tubs, water tanks, roofing, pipes, cladding and external door skins. Details are given in Mayer (1993) or East Coast Fibreglass Supplies (2010). Fig. 1(a) shows the original gray value image of the glass fiber reinforced polymer sample, provided by Velthuis from the IVW in Kaiserslautern. The image was recorded by Rack and Goebbels at the synchrotron BESSY in Berlin with a pixel sampling of 3.5 μm. As shown in the surface rendering of the binarized image Fig. 1(b), fibers do exhibit some small level of curvature.



(a) 2D sections of the original gray-value image along the xy (left) and xz planes (right), shown at the same scale.



(b) Surface rendering of the binarized image.

Fig. 1. GRP sample from R. Velthuis (IVW Kaiserslautern) recorded by Rack and Goebbels at the BAMline (BESSY II, Berlin, Germany) with a pixel sampling of 3.5 μm.

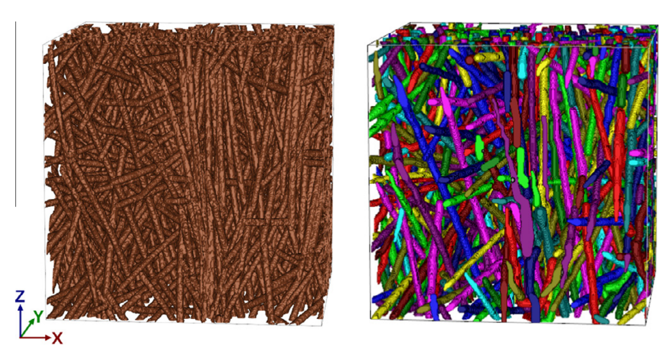
2.2. Analysis of the microstructure characteristics

The main geometrical and statistical characteristics of the fibrous microstructure, important for this study, are the volume fraction and the distributions of the fiber orientations, radii, lengths and curvatures. We measure the fiber volume fraction from the binarized image. Use is made of local analysis techniques to determine the fiber radius and orientation, that are directly applied on the gray value images (Altendorf and Jeulin, 2009a). Still, characteristics as lengths and curvature necessitate a fiber tracking approach, that makes it possible to follow the path of single fibers. In this respect, an algorithm for single fiber tracking is proposed in Altendorf and Jeulin (2009b).

Fig. 2 shows surface renderings of the reconstructed and labeled fibers versus the surface rendering of the original sample. Although most fibers are correctly reconstructed, it happens that fibers are split in two parts. This effect influences only the length estimation. Furthermore, the fact that most fibers extend over both boundaries of the image, complicates the length estimation. As such, most techniques used to estimate the average fibers length are not reliable, independently of the reconstruction.

Fig. 3 shows the length-weighted radius distribution from the separated fibers. The term length-weighted is to be understood as weighted statistics proportional to the fiber lengths, i.e. long fibers contribute more than short fibers. It is reminded that the number or length-weighted radius distribution is more accurate than the volume-weighted version (Altendorf et al., 2012), as the volume is dependent on the radius and therefore the volume-weighted radius distribution is distorted towards larger radii. The normal distribution $\mathcal{N}(5.44, 0.59)$ fits well to the numerically estimated radius distribution as shown in Fig. 3.

Fig. 4 shows the length-weighted orientation distribution of the fiber system on the unit sphere from two view angles. We observe a high probability of the direction along the z axis and a faint increase on a girdle passing the z axis. This distribution structure is fitted by a mixture of two β orientation distributions: one with preferred orientation close to the z axis and a second independent girdle distribution (Altendorf, 2011, Section 8.3). The β-distribution (Schladitz et al., 2006; Ohser and Schladitz, 2009) is a non-directed orientation distribution with one global parameter $\beta \in \mathbb{R}^+ \setminus \{0\}$. For $\beta = 1$ it results in the uniform distribution on the sphere, for $\beta \to 0$ the distribution concentrates on the z-axis and for $\beta \to \infty$



(a) Binary image. (b) Separated fibers.

Fig. 2. Surface label renderings of the original binary image and the separated fibers.