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Finite element study for conical indentation of elastoplastic micropolar material

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ABSTRACT

Numerous experiments have repetitively shown that the material behavior presents effective size dependent mechanical properties at scales of microns or submicrons. In this paper, the size dependent behavior of micropolar theory under conical indentation is studied for different indentation depths and micropolar material parameters. To illustrate the effectiveness of the micropolar theory in predicting the indentation size effect (ISE), an axisymmetric finite element model has been developed for elastoplastic contact analysis of the micropolar materials based on the parametric virtual principle. It is shown that the micropolar parameters contribute to describe the characteristic of ISE at different scales, where the material length scale regulates the rate of hardness change at large indentation depth and the value of micropolar shear module restrains the upper limit of hardness at low indentation depth. The simulation results showed that the indentation loads increase as the result of increased material length scale at any indentation depth, however, the rate of increase is higher for lower indentation depth, relative to conventional continuum. The numerical results are presented for perfectly sharp and rounded tip conical indentations of magnesium oxide and compared with the experimental data for hardness coming from the open literature. It is shown that the satisfactory agreement between the experimental data and the numerical results is obtained, and the better correlation is achieved for the rounded tip indentation compared to the sharp indentation.

1. Introduction

Nanoindentation is one of the most suitable means of testing the mechanical properties of materials at the nano scales for their economic as well as precise and nondestructive feature. However, there are numerous indentation test reports that have shown the measured hardness increases significantly with decreasing the indentation depth or the indenter size (Elmustafa and Stone, 2002; Feng and Nix, 2004; Gouldstone et al., 2007; Huang et al., 2006; Ma and Clarke, 1995; Pharr et al., 2010; Qiao et al., 2010). On the basis of established ISE model of Nix and Gao (1998), there is a linear relation between the square of indentation hardness and the inverse of indentation depth. While this relation is in agreement with microindentation, several other nanoindentation results show that the Nix–Gao model cannot predict the indentation hardness at nano scale and it overestimates the hardness of polycrystalline materials such as MgO at nano scale (Elmustafa and Stone, 2002; Feng and Nix, 2004; Huang et al., 2006; Pharr et al., 2010). Furthermore, numerous experiments have repetitively shown that the material behavior presents effective size dependent mechanical properties at scales on the order of a micron or a submicron, such as the micro-torsion of thin copper wires (Fleck and Hutchinson, 1993), the micro-bending of thin nickel foils (Idiart et al., 2009), compression of micropillar (Greer et al., 2005), and particle-reinforced metal–matrix composites (Nan and Clarke, 1996). These motivate the development that is able to bridge the gap between conventional continuum theories and size dependent material properties.

Due to the lack of internal material lengths in the constitutive model, this size dependence observed at the micron or sub-micron scale cannot be explained by classical continuum theory. At these scales, however, there are still hundreds of dislocations such that the collective behavior of these dislocations can be described by implementing continuum plasticity theory (but not classical plasticity). Among the continuum theories involving higher order gradients of the displacement, the micropolar (Cosserat) and the micromorphic theories are continuum models which take into account the material microstructure (Forest and Sievert, 2006; Grammenoudis and Tsakmakis, 2009). In the micropolar theory, three micro-rotations are introduced in addition to the classical displacements at each material point. This leads to a non-symmetric

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stress (strain) and a high order couple stress (torsion). Early Models for micropolar plasticity are formulated by Green et al. (1968), Lippmann (1969), Besdo (1974), and Muhlhaus and Vardoulakis (1987). This theory has been applied to study the strain-softening material response and the strain localization (de Borst, 1993; Dietsche et al., 1993; Ebrahimian et al., 2012; Khoei and Bakhshiani, 2005; Sharbati and Naghdabadi, 2006). De Borst (1993) generalized the J_2 flow theory with the couple stress and the micro curvature. The more general elastoplastic micropolar models in the large deformation range was developed by Forest et al. (1997) and Grammenoudis and Tsakmakis (2005, 2009). Altenbach et al. (2010) presented a review on Cosserat-type models of plates and shells. Ramezani and Naghdabadi (2010) generalized the hypo-elasticity for the micropolar media. A constitutive model for finite deformation micropolar plasticity exhibiting kinematic and isotropic hardening has been proposed by Grammenoudis and Tsakmakis (2009).

Recently, the finite element model of Cosserat materials for 2D elastic–plastic contact (Zhang et al., 2012) and 3D elastic contact (Xie et al., 2012) have been developed based on the parametric virtual work principle. They have shown that the rotation raised by polar characteristics produces a significant impact on stress field of an elastoplastic cylinder contacting with an elastic plate. Their numerical results showed that there are essential differences of contact characteristics between micropolar and classic models. In this study, we will show that Zhang's finding for the contact mechanism of the elastoplastic micropolar material is contributed to the indentation size effect, in which the presence of micropolar media promotes higher plastic hardening right under the indenter tip, and consequently leads to increase material hardness at low indentation depths.

Up to now, the size dependent behavior of micropolar plasticity theory is not fully discovered by the indentation simulations. The purpose of this paper is to develop and implement a finite element model for studying micro and nano-indentation of the elastoplastic micropolar material. In this paper, the elastic part of the constitutive model is based on the Eringen's (1999) micro-continuum field theory. De Borst's (1993) generalized yield criteria for the micropolar plasticity has been used to determine the effect of the material length scale on predicted hardness for small conical indents. The indentation contact problem is, of course, a very important aspect of the study that have been modeled by Zhang's et al. (2012) method for the contact analysis of multiple Cosserat bodies. The contact problem of indentation between the indenter and the elastoplastic micropolar surface is described by Coulomb's frictional law. To illustrate the effectiveness of the micropolar theory in predicting the indentation size effect (ISE), an axisymmetric finite element model has been developed for elastoplastic contact analysis of micropolar materials. The numerical results are presented for the micro and nano-indentation of magnesium oxide (MgO) with perfectly sharp and rounded tip conical indenters, and they are compared with the experimental hardness data of Feng and Nix (2004). The significance of MgO ISE has been discussed by Huang et al. (2006), Qiao et al. (2010) and Pharr et al. (2010) where it has different ISE characteristics at micro and nano scale and Nix–Gao model breaks down at small scales.

2. Theory

2.1. Micropolar elastoplasticity

The motion of a micropolar continuum is defined by two independent motions which are determined by; the macro displacement (u_j) and the micro rotation (φ_j). The strain and curvature tensors are defined as follows:

$$\varepsilon_{ij} = \frac{\partial u_j}{\partial x_i} - \epsilon_{ijk}\varphi_k, \quad \kappa_{ij} = \frac{\partial \varphi_j}{\partial x_i} \tag{1}$$

With expanding the free energy into the Taylor series in the vicinity of the natural state, disregarding the terms of higher orders, for isotropic, homogeneous, and centrosymmetric bodies, we obtain the following form of the elastic constitutive equations:

$$\sigma_{ij} = \frac{\partial F}{\partial \varepsilon_{ij}} = D_{ijkl}\varepsilon^e_{kl}, \quad m_{ij} = \frac{\partial F}{\partial \kappa_{ij}} = G_{ijkl}\kappa^e_{kl} \tag{2}$$

where σ_{ij} and m_{ij} are the components of force and moment stress tensors. D_{ijkl} and G_{ijkl} are fourth-order linear elastic micropolar modulus tensors:

$$\begin{aligned} D_{ijkl} &= \mu(1+\alpha)\delta_{ik}\delta_{jl} + \mu(1-\alpha)\delta_{il}\delta_{jk} + \lambda\delta_{ij}\delta_{kl}, \\ G_{ijkl} &= 4\mu\ell^2(\delta_{ik}\delta_{jl} + b\delta_{il}\delta_{jk} + c\delta_{ij}\delta_{kl}) \end{aligned} \tag{3}$$

Therein, μ and λ are the Lame's constants, whereas α ,b ,c and ℓ are the additional parameters of the micropolar theory. Hence, α is micropolar shear module which governs the influence of the skew symmetric part of the elastic micropolar strain and ℓ can be interpreted as an intrinsic length scale. The response of the body is influenced heavily with the ratio of the characteristic length (associated with the external stimulus) to the intrinsic length scale.

Let the elastic strain rate be given by the elastic constitutive equation, making use of the additive decomposition of general strain and curvature tensors increment into elastic and plastic parts; we obtain the following elastoplastic micropolar constitutive equations:

$$d\sigma_{ij} = D_{ijkl}(d\varepsilon_{kl} - d\varepsilon^p_{kl}), \quad dm_{ij} = G_{ijkl}(d\kappa_{kl} - d\kappa^p_{kl}) \tag{4}$$

The flow rule associated with the micropolar yield function has the general form:

$$d\varepsilon^p_{ij} = \lambda\frac{\partial f}{\partial \sigma_{ij}}, \quad d\kappa^p_{ij} = \lambda\frac{\partial f}{\partial m_{ij}}, \ \lambda \geqslant 0, \ f \leqslant 0, \ \lambda f = 0 \tag{5}$$

Here, f is a plastic potential function which is differentiable with respect to the stresses. The plastic multiplier λ is determined from the loading conditions. In optimization theory, such a set of equations is called a Kuhn–Tucker–Karush condition. According to the isotropic hardening model, the micropolar yield function is generally expressed as

$$f(\sigma_{ij}, m_{ij}, \varepsilon^p_{ij}, \kappa^p_{ij}, \gamma) \leqslant 0 \tag{6}$$

where γ is the hardening variable. By expanding micropolar yield function in a Taylor series, neglecting second and higher order terms, substituting the elastoplastic micropolar constitutive equation (Eq. (4)) and the flow rule (Eq. (5)) in a Taylor series, and introducing the slack parameter, the inequality form of micropolar yield criteria can be expressed by following the linear complementarity problem:

$$f^0 + w^\sigma_{kl}d\varepsilon_{kl} + w^m_{kl}d\kappa_{kl} - M\lambda + \upsilon = 0, \quad \lambda\upsilon = 0, \ \lambda, \ \upsilon \geqslant 0 \tag{7}$$

Here, f^0 denotes the value of yield function at current state, υ is slack parameter, and:

$$w^\sigma_{kl} = \frac{\partial f}{\partial \sigma_{kl}}D_{ijkl}, \quad w^m_{kl} = \frac{\partial f}{\partial m_{kl}}G_{ijkl}, \tag{8}$$

$$M = \left(\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl} + \frac{\partial f}{d\varepsilon^p_{kl}}\right)\left(\frac{\partial g}{\partial \sigma_{kl}}\right) + \left(\frac{\partial f}{\partial m_{ij}}G_{ijkl} + \frac{\partial f}{d\kappa^p_{kl}}\right)\left(\frac{\partial g}{\partial m_{kl}}\right) - \frac{\partial f}{\partial \gamma}h$$

It is widely accepted that geometries of Berkovich or Vickers indenters are approximated with axisymmetric models by choosing the cone angle such that the projected area/depth of the two-dimensional cone is the same as the actual indenter