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Buckling and post-buckling of gradient and nonlocal plasticity columns experiencing softening

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ABSTRACT

The buckling and the post-buckling behaviors of a perfect axially loaded column are analytically investigated through a global bilinear moment–curvature elastoplastic constitutive law. Three plasticity cases are studied, namely the linear hardening plasticity law, the perfect elastoplastic case and the softening case. The applications of such a study can be found in various structural engineering problems, including reinforced concrete, steel, timber or composite structures. It is analytically shown that for all kinds of elastoplastic behaviors, the plasticity phenomena lead to a global softening branch in the load–deflection diagram. The propagation of the plasticity zone during the post-buckling process is analytically characterized in case of linear hardening or softening plasticity laws. However, it is shown that the unphysical elastic unloading solution necessarily occurs in presence of local softening moment–curvature constitutive law. A nonlocal plasticity moment–curvature softening law is then used to control the localization branch in the post-buckling stage. This nonlocal plasticity law includes the explicit and the implicit gradient plasticity law. Higher-order plasticity boundary conditions are derived from an extended variational principle. Some parametric studies finally illustrate the main findings of this paper, including the plasticity modulus effect on the post-buckling behavior of these plasticity structural systems.

1. Introduction

This paper is focused on the buckling and post-buckling behavior of elastoplastic columns with hardening or softening moment–curvature constitutive law. Softening is understood in this study as a decrease of the stress variable (or the bending moment variable at the beam scale) for an increase of the strain variable (or the curvature at the beam scale). This phenomenon typically arises when the material loses its strength during a degrading loading process. A lot of engineering applications are concerned by such kind of elastoplastic moment–curvature models, which may be used for characterizing the ultimate behavior of inelastic structural elements.

Softening moment–curvature laws were probably first introduced for modeling cracking phenomena in reinforced concrete beams (Wood, 1968). Such engineering bending moment–curvature models can be useful for the fundamental understanding of the collapse of structural members. For instance, these beam models can be used to compute the global behavior of structural members composed of quasi-brittle materials experiencing some material softening phenomenon beyond a critical threshold (reinforced concrete members, timber beams, composite members… see for instance Wood (1968), Bažant (1976), Jirásek and Bažant (2002), Bažant and Cedolin (2003), Casandjian et al. (2013) and Hellesland et al. (2013)). On the other hand, geometrical softening may also be modeled in a simplified unidimensional approach, with such a bending-curvature constitutive law. This geometrically nonlinear softening phenomenon is associated in this case with the local buckling phenomenon of thin walled structured. For instance, the plastic buckling of tubes in bending, coupled with the so-called ovalization phenomenon, can be modeled with such a hardening–softening moment curvature relationship (Calladine, 1982; Kyriakides and Ju, 1992; Yu et al., 1993; Reid et al., 1998; Kyriakides et al., 2008; Poonaya et al., 2009). The bending response of steel thin-walled members can also experience a softening stage induced by the local buckling phenomenon (Mazzolani and Gioncu, 2002). The localization process in these hardening–softening structural members has been already investigated in details for bending problems, but the coupling between softening and second-order geometrical effects at the beam scale has not been investigated in details in the literature, to the author's knowledge, at least for plasticity structural systems.

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To allow for analytical calculations, a linear hardening/softening modulus is considered in this paper for the bending moment curvature constitutive law. This is in concordance with the model of Wood (1968) in case of linear softening applied to reinforced concrete members. Vaz and Patel (2007) also studied a bilinear bending moment–curvature law for flexible pipes applications, with a plasticity hardening branch. Linear hardening moment–curvature can be also deduced at the beam cross section level from local stress–strain relationship, as shown for instance by Casandjian et al. (2013).

The bending behavior of such elastoplastic systems is well studied in the literature. In particular, it is shown since the seminal work of Wood (1968) that a local elastoplastic softening moment–curvature constitutive law leads to an unphysical elastic unloading solution. It is now well accepted that a nonlocal model including some additional length scales, has to be considered for softening media (and also for softening beams). For instance, Pijaudier-Cabot and Bažant (1987) developed a nonlocal Continuum Damage Mechanics model to control the localization process in the softening range. A lot of numerical results have been performed for such nonlocal structural elements (see for instance Jirásek and Bažant (2002)) but very few analytical results are available for bending or buckling of nonlocal inelastic softening elements. Such reference solutions are useful for a better understanding of the deep scenario of the localization process induced by the softening part of the constitutive law.

Nonlocal plasticity can be implemented in a gradient-based or integral-based version. The pioneer solutions for the gradient plasticity models were probably first elaborated by Mühlhaus and Aifantis (1991) and de Borst and Mühlhaus (1992) for a homogeneous bar under uniaxial loading, exhibiting some specific periodic localized solutions. Later, Challamel (2003) developed a gradient plasticity moment–curvature model, and obtained some similar localization solutions in case of uniform bending, controlled by the beam length scales. Challamel et al. (2008) generalized these solutions for the non-uniform bending of gradient plasticity or nonlocal-based softening beams. Challamel et al. (2010) also shown the link between gradient plasticity and nonlocal integral-based plasticity, in an archetypal elastoplastic hardening–softening beam. Polizzotto (2007) obtained some other kinds of solutions for a gradient plasticity model with hardening. The paper of Peerlings (2007) should be also mentioned for a theoretical analysis of a gradient plasticity beam under uniform bending. More recently, Jirásek et al. (2013) obtained some new solutions for the axial behavior of non-homogeneous gradient plasticity softening bars.

If the bending behavior of nonlocal elastoplastic beam systems has been now well investigated, the stability behavior of such enriched elastoplastic systems is probably less studied. This paper aims at developing a rational analysis of the buckling and post-buckling behavior of some straight columns, modeled by an elastoplastic hardening or softening moment–curvature model. A typical application concerned by this model is the stability behavior of reinforced concrete column. The coupling between material instability (associated with the local softening behavior) and structural stability has not been exhaustively explored for such structural problems. It is worth mentioning that the buckling and post-buckling behavior of a column with a bilinear moment–curvature law (with positive hardening) has been studied by Vaz and Patel (2007) in a geometrically exact framework. More recently, Challamel and Hellesland (2013) investigated the buckling and post-buckling behavior of nonlocal Continuum Damage Mechanics columns, and used asymptotic methods to highlight the specific imperfection sensitive phenomenon.

In this paper, the buckling behavior of a clamped column with a free end (cantilever column) is investigated. The column is modeled by an elastoplastic moment–curvature law with or without gradient terms. The gradient terms have been introduced for regularizing the softening problem which would have been not well-posed without this enriched constitutive law. The column is assumed to be homogeneous with a length L and with a constant cross section. An axially centered load P acts at the top of the perfect column (no initial imperfection for this problem). The deflection of the column with respect to its fundamental state is denoted by $w(x)$ (see Fig. 1)

2. Differential equations

2.1. Governing equation

We first start from the weak form of the equilibrium equations via the principle of virtual work:

$$\int_0^L M\delta w'' - Pw'\delta w' = 0 \tag{1}$$

leading to the direct equilibrium equations:

$$M'' + Pw'' = 0 \tag{2}$$

It is assumed that the behavior of the column is ruled by an elastoplastic moment–curvature relationship with a linear hardening (or linear softening), see Fig. 2. In case of softening, the gradient plasticity terms are added for restoring the well-posedness of the evolution problem (if the regularized problem still remains well-posed).

The constitutive law at the cross section level may be written from the bending moment–elastic curvature relationship:

$$M = EI(w'' - \chi_p) \tag{3}$$

where the curvature χ is related to the second-order derivative of the deflection $\chi = w''$, χ_p is the plastic curvature, and EI is the elastic bending stiffness.

The loading function combines a gradient plasticity model with a nonlocal integral-based plasticity model and is given by the following differential equation:

$$M - l_c^2 M'' = M_p + k_p\left(\chi_p + a^2\chi_p''\right) \tag{4}$$

This loading function depends on two length scales, namely l_c and a. The gradient plasticity model is found for the specific case $l_c = 0$. M_p is the plastic moment; k_p positive corresponds to a positive hardening behavior, whereas a negative value of k_p corresponds to a softening behavior. The perfect elastoplastic behavior is associated with a vanishing plastic modulus $k_p = 0$. Such kind of nonlocal plasticity laws has been successfully used by Challamel et al. (2008) or Challamel et al. (2010) for accurate computation of the bending collapse of elastoplastic beams. This model has been shown to be cast in a so-called micromorphic plasticity framework (see Forest (2009) or Challamel et al. (2010)). It is known that nonlocal plasticity can accurately control the post-failure process in softening three-dimensional media (Jirásek and Bažant, 2002). It has been shown that the nonlocal plastic loading function considered in Eq. (4) can be used for softening moment–curvature, in order to avoid localization in infinitely small areas along the beam (Challamel, 2008). Such kind of implicit gradient dependent yield condition is also generally considered by Aifantis (2011) for other applications at the material scale.

2.2. Elastic buckling and elastoplastic post-buckling

For sufficiently small curvatures after the buckling of the elastoplastic column, the column remains in its elastic phase, meaning