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Multi-scale constitutive modeling of Ceramic Matrix Composites by Continuum Damage Mechanics

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ABSTRACT

The microscale damage mechanisms in brittle ceramics are investigated in detail and a Continuum Damage Mechanics (CDM) model is developed in this work to study two common failure modes in Ceramic Matrix Composites (CMC), i.e. *matrix/interphase fracture* and *fiber sliding*. In order to empower the developed framework for performing crashworthiness studies, the effect of the dynamic energy density content on the microscale fracture modes of CMCs is also considered. The CDM model is developed within a physically consistent framework that includes basic fracture mechanics of CMCs. Also the CDM model is developed in such a way that most of the material parameters are directly obtainable form the experimental data rather than cumbersome and time consuming numerical curve fitting techniques. In order to construct a computationally effective multiscale analysis platform for CMCs, this work aims to provide an asymptotic solution for a microscale representative volume element (RVE) which represents the fiber, interphase and matrix interactions. The developed asymptotic solution can capture the non-linear response of CMCs through CDM model; and it considerably reduces the computational cost of hierarchical multiscale analysis in comparison to the numerical methods, e.g. numerical models that simulate the real microstructure. The CDM model and the RVE asymptotic solution are utilized to study the microscale damage mechanisms in CMC systems. It is shown that the developed scheme performs quite well in capturing available experiments in the literature and provides a comprehensive description of microscale damage mechanisms in CMCs. The developed framework can be utilized in the future developments of the hierarchical multiscale analysis of CMC systems.

1. Introduction

Achieving high mechanical properties and long service lifetimes in severe environments for Ceramic Matrix Composites (CMC), which is defined as ceramic fibers embedded in a ceramic matrix, require a subtle design and analysis of the deformation and damage mechanisms. The CMC structures display lower matrix failure strain compared to their fibers, and as a result the ceramic matrix undergoes micro-scale damage mechanisms prior to the fiber failure. Although both matrix and fiber are brittle material, CMCs show a non-linear stress–strain behavior under tensile loading that is quite uncommon for ceramics. It is well-understood in the literature that non-linear response of CMCs is an efficient means of redistributing stress and eliminating stress concentrations (Evans, 1995). This non-linear stress–strain response and ductile damage tolerable property of CMCs are ascribed to the fiber–matrix bonding properties. The strength and performance of the fiber reinforced CMCs are enhanced via the use of an interphase between the ceramic-fiber and the matrix. The interphase has several key functions in CMCs, including crack deflection, load transfer, diffusion barrier and residual stress relaxation (Naslain et al., 1995). The interphase medium may be constructed form layered crystal structure (PyroCarbon), multilayered structures (PyC-SiC), or porous materials (rare-earth phosphates), and they are applied onto the surface of the ceramic-fiber after removing the sizing through the Chemical Vapor Deposition (Buet et al., 2014). The interphase's functions are twofold, first it should provide a weak bonding between matrix and fiber so that the cracks are arrested by the interphase. Secondly the interphase acts as

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the load transfer medium between the matrix and the fiber, as in any fiber-reinforced composite, which supposes conversely a strong enough bonding.

The failure of the bonding in fiber reinforced CMCs is usually categorized into two classes associated with fracture of the fiber/matrix interface (partial debonding) and fiber slip (fiber pull-out). The *fracture mode* involves continuum damage and degradation of the interphase layer. The fiber sliding is expected to occur when a shear stress greater than the interfacial shear resistance is applied within the interphase layer. It is worthwhile noting that for debonding and sliding to occur, rather than brittle fiber fracture, the interfacial debonding energy, i.e. Γ_i, shall not exceed the fracture energy of the fiber, i.e. Γ_f. As discussed by He and Hutchinson (1989), in the case of small elastic mismatches the interface medium acts as a mechanical fuse when $\Gamma_i/\Gamma_f < 0.25$. In terms of experimental measurements, different tests were suggested to measure the interface parameters, the most commonly used being: (i) the push-through test performed with a flattened diamond tip, which is applied under an increasing load to the fiber end in a composite thin foil that is cut perpendicular to the fiber axis (Marshall, 1984), and (ii) tensile tests with unloading–reloading hysteresis loops, performed on 1D model composites in the non-linear stress–strain domain (Lamon et al., 1995).

From the above discussion, it appears that the fiber and matrix bonding strength should be neither too strong nor too weak. These contradictory requirements in the design of high performance CMCs, make the design of CMCs an extremely difficult materials science challenge, especially in the case of designing aerospace grade CMCs that are being used in advanced jet engines in which CMCs must reliably work in a severe environment (Naslain, 1998; Pompidou and Lamon, 2007).

Modeling approaches for the fracture prediction in ceramics are vast. The flaw size and toughening effects on the strength of the ceramics have been studied by McMeeking and Evans (1982) and Evans (1995) who have proposed the resistance-curves for the dynamic fracture of CMCs. The resistance-curves provide mechanism based descriptions for the stable fracture of large flaws, unstable crack growth of small flaws, and the transition region. Hutchinson and co-workers have successfully developed constitutive models for CMCs and have implemented them in finite-element codes (Xia et al., 1993; Xia and Hutchinson, 1994). Moiré Interferometry experiments by Genin and Hutchinson (1997) have shown the difference between CMC loaded in tension and shear where shear band localization occurs. Holmquist and Johnson (2005) and Holmquist and Johnson (2008) link the inelastic deformation of ceramics to microcrack formations. Damage mechanics of CMC have also been widely studied in the literature. For example, Talreja (1991) discusses a thermodynamics based formulation of constitutive relationships with internal damage variables to derive the stress–strain-damage relationships. High temperature damage mechanisms of CMCs are studied by Sørensen et al. (1993), Maire and Chaboche (1997) and Maire and Lesne (1997). Clayton and co-worker have study the mesoscale modeling of dynamic fracture of ceramics (Clayton, 2005; Clayton et al., 2012).

Simulation of progressive fracture in composites encounters with a few computational challenges including stress singularity at the crack tip, ill-posed constitutive relation due to the crack tip localization effects, and estimating the correct path and length for each crack propagation. Several computational methods have been developed during past decades to address these milestones, including Cohesive Zone Model (CZM), Extended Finite Element Model (XFEM), and Phase Field Model (PFM). One may notice that one of the major limitations for CZM approach is that the crack propagation path needs to be known as a priori to lay specialized *cohesive elements* within that path to simulate the fracture (Ouyang and Li, 2009a,b; Ji et al., 2010). Thus, a random tortuous crack path is usually replaced by a straight line crack. Also the CZM material parameters suffer from mesh dependency. XFEM approach, proposed by Belytschko and Black (1999) and Song et al. (2006), utilizes enriched elements in which cohesive properties of the material system controls the degeneration and splitting of the enriched elements (Fries and Belytschko, 2010). The PFM method, introduced by Cahn and Hilliard (1958), considers the damage as a phase transformation process in the material system and provides constitutive description for the evolution of the microvoids and microcracks (Voyiadjis and Mozaffari, 2013). The multiscale CDM framework in this work has the following features (Shojaei and Li, 2014; Shojaei et al., 2014):

(1) The CDM is formulated based upon the damage dissipation energy and the fracture path evolves naturally based on fracture and thermomechanical properties of the CMCs.
(2) For finite element analysis (FEA) implementation purposes, there is no need for specialized elements, such as *cohesive element*; and CDM utilizes the conventional elements available in standard FEA packages.
(3) The CDM can be coupled with plasticity constitutive relations in order to study the elasto-plastic-damage response of the CMC system.
(4) In the case of highly localized problems, such as crack tip stress–strain fields, a strong mesh dependency is observed in FEA results, which is due to the *ill-posedness* of the elasticity constitutive relations (de Borst and Muhlhaus, 1991; de Borst and Sluys, 1991; Shojaei et al., 2013). The dissipated energies within these localized fields decrease upon the mesh refinement steps. This issue is usually alleviated by introducing a characteristic length into the CDM formulation. The energy dissipated during the damage process is specified per unit area, not per unit volume (ABAQUS, 2013). Hence, the damage dissipated energy is treated as an additional material parameter and it is used to compute the displacement at which full material damage occurs. This formulation ensures that the correct amount of energy is dissipated and greatly alleviates the mesh dependency. Consequently the softening response of the constitutive law is expressed based on a stress–displacement relation, in which the displacement is computed from the energy descriptions, e.g. plastic and damage dissipation energies, instead of the ill-posed constitutive relations.

Despite recent progress on the modeling of fracture in CMC, there remains a need for an alternative Continuum Damage Mechanics (CDM) formulation to capture its dynamic fracture response in a physically consistent manner. It is worth noting that nearly all micromechanical models of CMC assume that their fiber–matrix interface has no thickness even though it typically ranges between 0.1 μm and 1 μm for fibers of diameter between 7 μm and 20 μm – see Naslain et al. (1995) and Naslain (1998). This paper presents a multi-scale model, formulated within a CDM framework, to investigate the effects of interface strength, material properties and fiber/interface size effects on the overall mechanical properties of CMCs. The CDM model is formulated within the fracture mechanics framework, and it takes into account the physics behind the microcracking process, interaction between microcracks and the effects of dynamic energy density. Predictions by the CDM model are compared to experimental data and they will be shown to be in good agreement.

The manuscript is organized as follows.

Section 2: The constitutive relation between *elastic strain* and *Cauchy stress* is introduced.