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Dynamic interaction of a pile with a transversely isotropic elastic half-space under transverse excitations

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ABSTRACT

This investigation is concerned with a mathematical analysis of an elastic circular cylindrical pile embedded in a transversely isotropic half-space under lateral dynamic excitations. A combination of time-harmonic horizontal shear force and moment are applied at the top end of the pile. The boundary value problem is formulated by decomposing the pile-medium system into a fictitious pile and an extended transversely isotropic half-space. A Fredholm integral equation of the second kind governs the interaction problem, whose solution is then computed numerically. Selected results for dynamic compliance bending moment, displacement and slope profiles are presented for different transversely isotropic half-spaces to portray the influence of degree of anisotropy of the medium on various aspects of the solution.

1. Introduction

The mechanical response of anisotropic materials under different external and internal loads has attracted increasing attention in recent years, chiefly because of the limitations in using isotropic theories to dealing with man-made materials such as composites and natural materials such as deposit soils and rocks. The study of load transfer in transversely isotropic media is one of the prominent research areas in this category, with relevance to a variety of subjects such as geomechanics and material science. On the analytical approaches for the solid mechanics boundary value problems, researchers have begun to address the influence of degrees of anisotropy. For instance, in the context of rigid inclusion and crack problems, Selvadurai (1980) investigated the asymmetric static deformation of an infinite transversely isotropic medium containing a rigid disc as inclusion. The corresponding cases of an elliptical inclusion, penny-shaped cracks have also been examined as in Selvadurai (1984), and Rahman (1999, 2002). The problem of a circular cylindrical inclusion in a transversely isotropic solid was also investigated by Hasegawa and Kisaki (2003). On the dynamic counterpart of the foregoing class of problems, Hanson and Puja (1997) and later Rahimian et al. (2006) studied a transversely isotropic half-space subjected to a forced constant amplitude torsional motion of a rigid circular disc bonded to the surface of the medium. The interaction problem of a rigid embedded disc under either static or time-harmonic loads in a transversely isotropic medium was studied in Katebi et al. (2008), Eskandari-Ghadi and Ardeshir-Behrestaghi (2010) and Eskandari-Ghadi et al. (2010, 2011, 2013) for different boundary conditions. Eskandari-Ghadi et al. (2013) have made a detailed analytical investigation for a circular crack buried in a transversely isotropic half-space subjected to mono-harmonic force in the first mode. They have degenerated their solution for a static case, and for a full-space in dynamic case. In the field of soil-structure interaction involving transversely isotropic media, several attempts have also been made. For instance, dynamic solutions for the displacement of a transversely isotropic stratified medium with a rigid base were reported by Waas et al. (1985). Wang and Rajapakse (2000) applied a boundary element method to study the dynamic response of rigid cylindrical and hemispherical foundations in transversely isotropic half-space. The same problem was later pursued by means of an approximate regularized indirect BEM by Barros (2003). Following the approach, Barros (2004) considered the soil–pile interaction problem in a transversely isotropic medium by replacing the pile as a series of one dimensional beam elements, while the soil response was formulated in an integral sense analogous to an indirect boundary element method. In his definition of soil's influence functions, fictitious force distributions over a cylindrical surface element were assumed to be constant depthwise in the vertical direction with a sinusoidal variation in the angular direction, and the coupling between the FEM and BEM models was enforced at the midpoint of the beam elements. As a result of the dissimilar discretization and representation of the pile and the soil, however,

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there is an inherent displacement incompatibility between the pile and the soil over the length of the embedment. Recently, Amiri-Hezaveh et al. (2013) have found the impedance functions in any direction for a rectangular rigid foundation bonded on a multi-layered transversely isotropic half-space, where they have analytically separated the effects of SH-wave from both P- and SV-waves.

Since the soil has to be considered as a three-dimensional continuous domain and the pile has to be treated at a minimum as a one–dimensional structural medium, rigorous analysis of soil–pile interaction has been difficult to achieve in applied mechanics. With the increasing recognition that the isotropic assumption for the soil may not be compatible with the conditions of many natural deposits as well, there is a need to address this class of load-transfer problems in the context of an anisotropic medium as well. By means of a variational boundary integral equation statement, for example, Rajapakse (1988,) suggested an approximate scheme to tackle a related bar-soil interaction problem, which was generalized in Rajapakse and Wang (1990) to account for some static load conditions. By assembling dynamic stiffness matrices of a discretized soil, Liu and Novak (1994) and later, Tiesheng (1997) presented a finite element model for the interaction problem of a pile and a transversely isotropic layered medium. Considering the transverse isotropy of the soil, an analytical method for transient torsional dynamic response of a pile embedded in a saturated medium was given by Chen et al. (2007). Wang et al. (2009) also considered an analogous study for an end-bearing pile.

Allowing more rigorous treatment of cylindrical embedments in the framework of classical elastostatics, Muki and Sternberg (1969, 1970) developed the idea of using an extended half-space with a one-dimensional fictitious reinforcement as an approximate representation of an axially-loaded cylindrical pile of finite or infinite length that is bonded to a hole in the medium. Extending the concept to the lateral pile problem involving an arbitrary combination of lateral horizontal forcing, shear force and bending moment in both static and dynamic conditions, Pak and Jennings (1987) and Pak (1989) showed that the asymmetric soil–pile interaction problem can be reduced to a compact Fredholm integral equation of the second kind. While exact formulations have since been developed in Pak and Ji (1993) and Abedzadeh and Pak (2004), the simplicity of Muki and Sternberg's extended half-space concept has attracted numerous followings for the isotropic solid or porous media (e.g., Apirathvorakij and Karasudhi, 1980; Flowler and Sinclair, 1978; Karasudhi et al., 1984; Rajapakse and Shah, 1987; Rajapakse and Shah, 1987; Selvadurai and Rajapakse, 1987; Zeng and Rajapakse, 1999). Among these studies, various attempts have also been made to improve the model. For instance, Rajapakse and Shah (1987,) and Pak (1989) adopt non-uniform body-force fields to modify the displacement compatibility of the fictitious reinforcement and the extended half-space. Using the fictitious bar-extended half-space model, Lagrange's equation concept and a set of exponentially decaying functions with respect to depth, Rajapakse and Shah (1987,) presented a numerical scheme for the boundary value problem. Comparing the results with prior discretization methods used by Sen et al. (1985) and Flowler and Sinclair (1978), Rajapakse and Shah (1987) stated that the foregoing methods do not properly account for inertia component of the bar-half space system. This deficiency is considered due to two major points: (1) The fact that the mass density cannot be negative. If the mass density is negative then the inherent mathematical properties of the equations of motion is lost; (2) The displacement incompatibility of the fictitious bar and the extended half-space. However, in their development, Rajapakse and Shah (1987,) modified the technique by assuming a non-uniform body force field acting on the extended half-space. Rajapakse and Shah (1987,) also introduced frequency ranges for each loading case wherein the use of fictitious bar-extended half-space model yields results with reasonable accuracy.

Aimed to provide a reference solution to the dynamic laterally-loaded pile problem in a transversely isotropic half-space, this paper is concerned with the development of a rigorous mathematical solution in the spirit of the extended medium approach. Following the compact and elegant formulation of Pak and Jennings (1987) and Pak (1989), a set of influence functions corresponding to a buried time-harmonic horizontal body-force field which can represent more closely the high sectional rigidity of a pile is employed to represent the soil reaction from low-frequency pile motions. The problem is mathematically reduced to one unknown that is governed by a single Fredholm integral equation of the second kind, thereby maximizing analytical consistency while minimizing multivariate discretization errors in many past numerical studies. Selective numerical results are presented and discussed to illustrate the material anisotropy effects on different aspects of the lateral dynamic soil–pile interaction problem, including the frequency-dependent pile-head compliance functions which are of most common engineering interest. As a verification of the solution, the results are degenerated for the static case and compared with the existing solutions for isotropic materials as in Pak and Jennings (1987) and Pak (1989).

2. Mathematical formulation

A mathematical formulation is proceeded in this section for the dynamic interaction problem. Following the procedure by Pak and Jennings (1987), we introduce $\{o; x_1, x_2, x_3\}$ as a rectangular Cartesian coordinate frame which spans 3D-space $\bar{E}$ with the unit based vectors $\vec{e}_1, \vec{e}_2$ and $\vec{e}_3$. In this investigation, a circular cylindrical elastic pile P of length l and radius a, which is partially embedded in a transversely isotropic half-space is under consideration (Fig. 1). The isotropic planes of the embedding medium are parallel to $x_1 - x_2$ plane with Young's modulus E and Poisson's ratio ν, and the elastic coefficients in any plane parallel to the x_3-direction, which is perpendicular to the isotropic planes are denoted by E', ν' and G', where G' is the shear modulus. In addition, the longitudinal centroidal axis of the pile is considered to be coincident with x_3-axis and a combination of time-harmonic lateral force $\vec{V} = V_0 e^{i\omega t}\vec{e}_1$ and bending moment $\vec{M} = M_0 e^{i\omega t}(-\vec{e}_2)$ with frequency ω are acting in the $x_1 - x_3$ plane at the top end of the pile.

By the foregoing description, the half-space can be denoted as $H = \{\vec{x} | \vec{x} \in \bar{E}, x_3 > 0\}$, the pile region occupied by the pile as $D = \{\vec{x} | \vec{x} \in \bar{E}, x_1^2 + x_2^2 < a^2, 0 < x_3 < l\}$ and the open cross section of the pile located at the depth $x_3 = s$ as $\Pi_s = \{\vec{x} | \vec{x} \in D, x_3 = s < l\}$. In the present treatment, as in Pak and Jennings (1987) and also

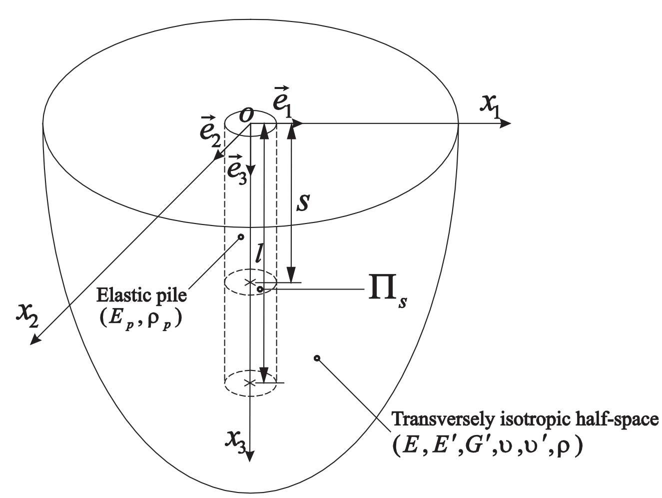

Fig. 1. Geometry of pile and embedding medium.