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Higher order model for soft and hard elastic interfaces

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ABSTRACT

The present paper deals with the derivation of a higher order theory of interface models. In particular, it is studied the problem of two bodies joined by an adhesive interphase for which "soft" and "hard" linear elastic constitutive laws are considered. For the adhesive, interface models are determined by using two different methods. The first method is based on the matched asymptotic expansion technique, which adopts the strong formulation of classical continuum mechanics equations (compatibility, constitutive and equilibrium equations). The second method adopts a suitable variational (weak) formulation, based on the minimization of the potential energy. First and higher order interface models are derived for soft and hard adhesives. In particular, it is shown that the two approaches, strong and weak formulations, lead to the same asymptotic equations governing the limit behavior of the adhesive as its thickness vanishes. The governing equations derived at zero order are then put in comparison with the ones accounting for the first order of the asymptotic expansion, thus remarking the influence of the higher order terms and of the higher order derivatives on the interface response. Moreover, it is shown how the elastic properties of the adhesive enter the higher order terms. The effects taken into account by the latter ones could play an important role in the nonlinear response of the interface, herein not investigated. Finally, two simple applications are developed in order to illustrate the differences among the interface theories at the different orders.

1. Introduction

Interface models are widely used for structural analyses in several fields of engineering applications. They are adopted to simulate different structural situations as, for instance, to reproduce the crack evolution in a body according to the cohesive fracture mechanics (Barenblatt, 1962; Needleman, 1990), to study the delamination process for composite laminates (Corigliano, 1993; Point and Sacco, 1996, 1998), to simulate the presence of strain localization problems (Belytschko and Black, 1999; Moës and Belytschko, 2002; Ortiz et al., 1987) or to model the bond between two or more bodies (Frémond, 1987; Xu and Wei, 2012). Interfaces are mostly characterized by zero thickness even when the physical bond has a finite thickness, as in the case of glued bodies. This physical thickness of the adhesive can also be significant, as in the case of the mortar joining artificial bricks or natural blocks in the masonry material.

Interface models have the very attractive feature that the stress defined on the corresponding points of the two bonded surfaces, $\boldsymbol{\sigma}\mathbf{n}$ with $\mathbf{n}$ unit vector normal to the interface, assumes the same value, $[\boldsymbol{\sigma}\mathbf{n}] = 0$, and it is a function of the relative displacement, $[\mathbf{u}]$:

$$\boldsymbol{\sigma}\mathbf{n} \leftrightarrow [\mathbf{u}], \tag{1}$$

where the brackets $[\,]$ denote the jump in the enclosed quantity across the interface.

As a consequence, the interface constitutive law is assumed to relate the stress to the displacement jump. This constitutive relationship can be linear or it can take into account nonlinear effects, such as damage, plasticity, viscous phenomena, unilateral contact and friction (Alfano et al., 2006; Del Piero and Raous, 2010; Parrinello et al., 2009; Raous, 2011; Raous et al., 1999; Sacco and Lebon, 2012; Toti et al., 2013). As a consequence, different interface models have been proposed in the scientific literature. Moreover, interface models are implemented in many commercial and research codes as special finite elements.

Interface models can be categorized into two main groups. In the first group, the interface is characterized by a finite stiffness, so that relative displacements occur even for very low values of

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interface stresses; in such a case, the interface is often named in literature as "soft":

$$[\mathbf{u}] = f(\boldsymbol{\sigma}\mathbf{n}), \quad [\boldsymbol{\sigma}\mathbf{n}] = 0. \tag{2}$$

On the contrary, in the second group of models, interfaces are characterized by a rigid response, preceding the eventual damage or other inelastic phenomena; the interface is called "hard" and for the linear case it is governed by the equations:

$$[\mathbf{u}] = 0, \quad [\boldsymbol{\sigma}\mathbf{n}] = 0. \tag{3}$$

The interface models in the first group are widely treated in literature, as they are governed by smooth functions and, consequently, they can be more easily implemented in finite element codes; moreover, inelastic effects can be included as in a classical continuum material. In this instance, the numerical procedures and algorithms are derived and implemented as an extension of the ones typical of continuum mechanics.

The models in the second group are less studied in literature; they are governed by non-smooth functions when nonlinearities are considered and they require the use of quite powerful mathematical techniques; moreover, finite element implementations are more complicated (Dumont et al., 2014).

A rigorous and mathematically elegant way to recover the governing equations of both soft and hard interfaces is represented by the use of the concepts of the asymptotic expansion method. This method was developed by Sanchez-Palencia (1980) to derive the homogenized response of composites; it is based on the choice of a geometrically small parameter (e.g. the size of the microstructure) and on the expansion of the relevant fields (displacement, stress and strain) in a power series with respect to the chosen small parameter. This technique was successfully used to recover the plate and shell theories (Ciarlet, 1997; Ciarlet and Destuynder, 1987) or the governing equations of interface models (Geymonat and Krasucki, 1997; Klarbring and Movchan, 1998; Lebon et al., 1997; Licht and Michaille, 1996; Marigo et al., 1998).

When the thickness of the bonding material, ε, is not so small, higher order terms in the asymptotic expansions with respect to ε should be considered in the derivation of the interface governing equations. Previous studies have established that, if the stiffness of the adhesive material is comparable with the stiffness of the adherents, then various mathematical approaches (asymptotic expansions (Abdelmoula et al., 1998; Benveniste, 2006; Benveniste and Miloh, 2001; Geymonat et al., 1999; Hashin, 2002; Klarbring and Movchan, 1998; Lebon et al., 2004), Γ-convergence techniques (Caillerie, 1980; Lebon and Rizzoni, 2010; Licht, 1993; Licht and Michaille, 1997; Serpilli and Lenci, 2008), energy methods (Lebon and Rizzoni, 2011; Rizzoni and Lebon, 2012)) can be used to obtain the model of perfect interface at the first (zero) order in the asymptotic expansion. At the next (one) order, it is obtained a model of imperfect interface, which is non-local due to the presence of tangential derivatives entering the interface equations (Abdelmoula et al., 1998; Hashin, 2002; Lebon and Rizzoni, 2010, 2011; Rizzoni and Lebon, 2012, 2013).

The aim of this paper is the derivation of the governing equations for soft and hard anisotropic interfaces accounting for higher order terms in the asymptotic expansion, being the zero order terms classical and well-known in the literature. While the terms computed at the order one for hard interfaces (Eq. (64)) have been derived previously (Lebon and Rizzoni, 2010, 2011; Rizzoni and Lebon, 2013), the terms computed at the order one for soft interfaces (Eq. (56)) represent a new contribution. A novel asymptotic analysis is presented based on two different asymptotic methods: matching asymptotic expansions and an asymptotic method based on energy minimization. In the first method, the derivation of the governing equations is performed by adopting the strong formulation of the equilibrium problem, i.e. by writing the classical compatibility, constitutive and equilibrium equations. The second method relies on a weak formulation of the equilibrium problem and it is an original improvement of asymptotic methods proposed in Lebon and Rizzoni (2010), because the terms at the various orders in the energy expansion are minimized together and not successively starting from the term at the lowest order. The asymptotic analysis via the energy method is useful to ascertain the consistency and the equivalence with the method based on matched asymptotic expansions. Indeed, a main result of the paper consists in showing that the two approaches, one based on the strong and the other on the weak formulation, lead to the same governing equations. In addition, the derivation of the boundary conditions for an interface of finite length is straightforward via the energy method, while these conditions have to be specifically investigated using matched asymptotic expansions (Abdelmoula et al., 1998). Finally, the weak formulation is the basis of development of numerical procedures, such as finite element approaches, which can be used to perform numerical analyses in order to evaluate the influence and the importance of higher order effects in the response of the interface.

Another original result of the paper is a comparison of the equations governing the behavior of soft and hard interfaces obtained at order zero with the ones obtained at the first order in the asymptotic expansions. The influence of the higher order terms and of the higher order derivatives on the interface response is also highlighted and their dependence on the elastic properties of the adhesive is determined. Notably, the effects taken into account by the higher order terms in the asymptotic analysis could play an important role in the nonlinear response of the interface, herein not investigated.

The analysis of the regularity of the limit problems and of the singularities of the stress and displacement fields near the external boundary of the adhesive are not considered in this paper. For the model of soft interface computed at order zero, these questions are considered in Geymonat et al. (1999). Finally, it should be emphasized that the present analysis considers planar interphases of constant thickness. Thin layers of varying thickness have been considered in Ould Khaoua (1995) and higher order effects in curved interphases of constant thickness have been studied in Rizzoni and Lebon (2013) only for the case of a hard material.

The paper is organized as follows. In Section 2, the problem of two bodies in adhesion is posed, the rescaling technique is introduced and the governing equations of the adherents and of the adhesive are written, together with the matching conditions. In Section 3, the interface equations are derived for both the two cases of an adhesive constituted of a soft and a hard materials, and higher order terms in the asymptotic expansion are considered. In Section 4, the variational approach to the derivation of the governing equations of the interface is presented. Section 5 is devoted to the comparison between the lower and the higher order for both soft and hard interface models. Finally, two analytical examples are presented, the shear and the stretching of a two-dimensional composite block, and the main results are discussed.

2. Generalities of asymptotic expansions

A thin layer B^ε with cross-section S and uniform small thickness $\varepsilon \ll 1$ is considered, S being an open bounded set in R^2 with a smooth boundary. In the following B^ε and S will be called interphase and interface, respectively. The interphase lies between two bodies, named as adherents, occupying the reference configurations $\Omega^\varepsilon_\pm \subset R^3$. In such a way, the interphase represents the adhesive joining the two bodies Ω^ε_+ and Ω^ε_-. Let $S^\varepsilon_\pm$ be taken to denote the plane interfaces between the interphase and the adherents