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Comparative study of continuum damage mechanics and Mechanics of Porous Media based on multi-mechanism model on Polyamide 6 semi-crystalline polymer

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ABSTRACT

The biphasic character of semi-crystalline polymer was modeled by the multi-mechanism (MM) constitutive relationships. Here, a comparative study between continuum damage mechanics (CDM) theory and Mechanics of Porous Media (MPM) approach, both related to the MM model, is performed. This comparison is based upon creep tests conducted on notched round bars made of PA6 semi-crystalline polymer to enhance a multiaxial stress state. For CDM model, the damage is classically described by a unique overall variable whereas the average of the local porosity at each phase level was considered for the MPM model. For each model, the optimization of the set of material's parameters was carried out by combining the overall behavior of notched specimens subjected to creep loading, as well as the local information such as the distribution of porosity. It is found that both CDM and MPM models, each coupled with MM model correctly describe the overall creep behavior of the notched specimen if two damage variables are used. Moreover the MM/MPM model is more relevant for predicting porosity distribution.

1. Introduction

The frequent use of polymers in engineering components requires reliable constitutive models to describe both their mechanical and damage behaviors. Among these materials, semi-crystalline polymers (SCP) exhibit high non linear mechanical response caused by their structural changes that necessitates the development of accurate constitutive models. These models should be based on deformation and damage mechanisms to analyze inelastic behaviors of structures made of SCP. During the two last decades, extensive research was accomplished on the investigation of the behavior of SCP materials. These works are of two kinds (i) constitutive models (ii) durability (damage mechanics and failure).

- Examples of studies devoted to analyze SCP stress–strain response includes the works of Dusunceli and Colak (2007), Drozdov and Christiansen (2008), Baudet et al. (2009), Bles et al. (2009), Drozdov (2010), Epee et al. (2011), Ricard et al. (2014). The large deformation level, the strain rate effect, the influence of the crystallinity ratio are factors that influence the SCP behavior, see for instance Danielsson et al. (2002), Drozdov (2010), Epee et al. (2011), Drozdov (2013), Abu Al-Rub et al. (2014).
- In addition to the factors enumerated above, SCP might contain initial voids that grow and coalesce during deformation and should be considered. Therefore, SCP are assumed to be porous media containing micro-voids in the undeformed state considered as damage. They are at the origin of failure by their coalescence during mechanical loading (Laiarinandrasana et al., 2010; Boisot et al., 2011). The durability of the SCP was in the focus of a second class of researches. The durability prediction requires a better understanding of the failure mode of structures made of SCP, see for instance Cotterell et al. (2007), Wang et al. (2010), Detrez et al. (2011), Frontini et al. (2012), Leevers (2012), Ricard et al. (2014), Abu Al-Rub et al. (2014).

Two broad approaches have emerged in the literature to predict failure of materials: continuum damage mechanics (CDM) theory and the Mechanics of Porous Media (MPM) concept. The first class of model is known to provide good predictions under shear tensile loading condition where typically low stress triaxialities are encountered. Whereas, according to Brunig et al. (2013), for

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instance, porous plasticity model type is more appropriate at high stress triaxialities.

- CDM theory describes the effects of growth on macroscopic variables. This approach has been extensively applied to metallic materials (Lemaitre and Desmorat, 2001; Hambli, 2001; Lemaitre et al., 2000; Chaboche et al., 2006; Saanouni, 2008; Boudifa et al., 2009; Ayoub et al., 2011). In the CDM approach, damage is modeled at the macroscopic level by means of thermodynamic variable that leads to elastic moduli degradation and may also affect the plastic behavior. This approach can be extended in order to include volumetric plastic strains that play important roles in the ductile fracture under high triaxialities (Brunig et al., 2013).
- MPM models such as that of Gurson–Tvergaard–Needleman (GTN), initiated by Gurson (1977) are based on the assumption that damage occurs at microstructural level due to micro-void nucleation, growth and coalescence. Void growth is linked to the plastic strain and the stress triaxiality level. This approach was adopted for instance by Pardoen and Hutchinson (2000), Besson and Guillemer-Neel (2003), Monchiet et al. (2008), Sai et al. (2011), Oral et al. (2012), Ognedal et al. (2014). Improvements was proposed in order to take into account the effects of low triaxiality during shearing (Nahshon and Hutchinson, 2008; Nielsen and Tvergaard, 2009; Tvergaard and Nielsen, 2010). Some difficulties may also be encountered by the porous plasticity models that are not able to correctly predict the crack propagation path (Hambli, 2001). Another limitation of the model is that nucleation of the voids does not occur under compression (Nahshon and Xue, 2009).

Comparative studies between the GTN porous plasticity models and the CDM theories have been performed for instance by Hambli (2001), Mkaddem et al. (2004), Pirondi et al. (2006), Li et al. (2011), Malcher et al. (2012), Andrade et al. (2014).

The present work is a continuation of previous studies devoted to propose reliable constitutive models that consider both mechanical and damage behavior of Polyamide 6 (PA6) and Polyamide 11 (PA11) subjected to tensile and creep loadings. These models belong to two distinct approaches (i) the so-called multi-mechanism (MM) formalism (Regrain et al., 2009; Regrain et al., 2009; Sai et al., 2011; Cayzac et al., 2013) (ii) the unified approach (mono-mechanism) (Laiarinandrasana et al., 2010). The first category is more appropriate for SCP since it allows the distinction of the two phases by means of the crystallinity index. Therefore, it provides local information such as, plastic strains, stresses and damage in each phase. Only multi-mechanism model will be then considered in the sequel. The MM formalism that considers both mechanical and damage behavior of SCP was used to study the void growth, the creep and tensile behavior and the damage localization in notched specimens. An attempt is then made here to enroll the same MM model to GTN and CDM theory.

The novelties in this work are two folds:

- Coupling the CDM theory and the MM model: to the authors' best knowledge this association has never been proposed before;
- For the two MM-associated models, it is proposed to consider the damage (i) as an intrinsic local variable related to each specific phase, (ii) as a unique overall variable over both phases.

The paper is organized in the following manner: Constitutive equations of the proposed MM models are detailed in section 2. To assess their reliability, the two MM models are compared with experimental data base taken from the works of Regrain et al. (2009), Cayzac et al. (2013) in Section 3. A selection strategy is developed with the aim to choose the more relevant model. A first selection of the models is performed by comparison to creep behavior of notched specimen in Section 3.1. The local responses of the remaining models are then analyzed in Section 3.2 to retain the more appropriate model(s). The local contribution of damage state at each phase level is critically commented in Section 3.3.

2. Modeling

Since SCP consist of amorphous and crystalline phases, MM approach is good candidate to describe the polymeric material as a composite material. Amorphous and crystalline phases are, respectively, mapped to a first mechanism referred to as '*a*' and a second mechanism referred to as '*c*'. The MM approach is intended here to describe the contribution of the amorphous phase and the crystalline phase to the inelastic behavior of SCP characterized by their crystallinity ratio *z*.

The use of a finite strain formulation through updated lagrangian formalism is needed to model large-strain deformation of polymer. The material behavior is based on Green–Naghdi transformation of the stress–strain problem into an "equivalent material referential". This kind of formulation can be applied to materials with tensorial internal variables without modifying the local evolution rules (Ladeveze, 1999). The model is described by:

$$\underset{\sim}{\boldsymbol{L}} = \dot{\underset{\sim}{\boldsymbol{F}}}\,\underset{\sim}{\boldsymbol{F}}^{-1} \qquad \underset{\sim}{\boldsymbol{D}} = \frac{1}{2}\left(\underset{\sim}{\boldsymbol{L}} + \underset{\sim}{\boldsymbol{L}}^{T}\right) \qquad \underset{\sim}{\boldsymbol{\Omega}} = \frac{1}{2}\left(\underset{\sim}{\boldsymbol{L}} - \underset{\sim}{\boldsymbol{L}}^{T}\right) \tag{1}$$

where $\underset{\sim}{\boldsymbol{F}}$ is the deformation gradient, $\underset{\sim}{\boldsymbol{L}}$ the rate of deformation, $\underset{\sim}{\boldsymbol{D}}$ the stretch rate and $\underset{\sim}{\boldsymbol{\Omega}}$ the rotation rate. The stretch rate tensor is transported into a local rotated referential following the expression:

$$\underset{\sim}{\dot{\boldsymbol{e}}} = \underset{\sim}{\boldsymbol{R}}^{T}\,\underset{\sim}{\boldsymbol{D}}\,\underset{\sim}{\boldsymbol{R}} \tag{2}$$

where the rotation tensor $\underset{\sim}{\boldsymbol{R}}$ is determined by the polar decomposition of the deformation gradient $\underset{\sim}{\boldsymbol{F}} = \underset{\sim}{\boldsymbol{R}}\,\underset{\sim}{\boldsymbol{U}}$. $\underset{\sim}{\boldsymbol{R}}$ and $\underset{\sim}{\boldsymbol{U}}$ describe respectively a pure rotation and a pure stretch tensor.

The integrated strain tensor is decomposed into both elastic and inelastic parts. Thanks to updated lagrangian formulation, constitutive relations can be expressed as in small strain hypothesis. Therefore, dealing with the elastic strain tensor is equivalent to a hypoelastic formulation in agreement with a Green–Naghdi stress rate. The stress measure is here the Cauchy stress $\underset{\sim}{\boldsymbol{\sigma}}$ obtained by using the conjugate stress $\underset{\sim}{\boldsymbol{S}}$ which results from the material behavior integration:

$$\underset{\sim}{\boldsymbol{\sigma}} = det^{-1}(\underset{\sim}{\boldsymbol{F}})\,\underset{\sim}{\boldsymbol{R}}\,\underset{\sim}{\boldsymbol{S}}\,\underset{\sim}{\boldsymbol{R}}^{T} \tag{3}$$

Under the small deformation hypothesis and using the assumption of uniform elasticity, the total strain can be decomposed into an elastic part and an inelastic one.

$$\underset{\sim}{\boldsymbol{\varepsilon}} = \underset{\sim}{\boldsymbol{\varepsilon}}^{e} + \underset{\sim}{\boldsymbol{\varepsilon}}^{in} \tag{4}$$

The assumption of uniform strain in the semi-crystalline polymers was also made in the works of Brusselle-Dupend and Cangémi (2008), Baudet et al. (2009), Shojaei and Li (2013). An other group of works considers that each phase has its own elastic strain (Bédoui et al., 2004; Zairi et al., 2011). The total inelastic strain is the average of the irreversible deformation of each phase:

$$\underset{\sim}{\boldsymbol{\varepsilon}}^{in} = (1-z)\,\underset{\sim}{\boldsymbol{\varepsilon}}^{a} + z\,\underset{\sim}{\boldsymbol{\varepsilon}}^{c} \tag{5}$$

where $\underset{\sim}{\boldsymbol{\varepsilon}}^{a}$ and $\underset{\sim}{\boldsymbol{\varepsilon}}^{c}$ stand for the inelastic strain in the amorphous phase and the crystalline phase respectively. Such a decomposition has already been applied in other works (Nikolov and Doghri, 2000; Ahzi et al., 2003; van Dommelen et al., 2003; Sheng et al., 2004; Colak and Krempl, 2005).