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# Finite deformation effects in cellular structures with hyperelastic cell walls

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ABSTRACT

Cellular solids are remarkably strong structures built from seemingly fragile materials. In order to gain new insight into the mechanical behaviour of these omnipresent materials, we analyse the deformation of seamless cellular bodies within the framework of finite strain elasticity and identify behaviours which are not captured under the small strain regime. Assuming that the cell walls are hyperelastic, we devise a mathematical mechanical strategy based on a successive deformation decomposition by which we approximate the large deformation of periodic cellular structures, as follows: (i) firstly, a uniformly deformed state is assumed, as in a compact solid made from the same elastic material; (ii) then the empty spaces of the individual cells are taken into account by setting the cell walls free. For the elastic structures considered here, an isochoric deformation that can be maintained in both compressible and incompressible materials is considered at the first step, then the stresses in this known configuration are used to analyse the free shape problem at the second step where the cell geometry also plays a role. We find that, when these structures are submitted to uniform external conditions such as stretch, shear, or torsion, internal non-uniform local deformations occur on the scale of the cell dimension. For numerical illustration, we simulate computationally the finite elastic deformation of representative model structures with a small number of cells, which convey the complexity of the geometrical and material assumptions required here. Then the theoretical mechanical analysis, which is not restricted by the cell wall material or number of cells, indicates that analogous finite deformation effects are expected also in other physical or computer models.



## 1. Introduction

Cellular solids, by contrast to compact materials, are two or three dimensional bodies divided into cells, the walls of which are made of a solid material capable of undertaking (large) elastic deformations without plastic failure or fracture. Due to their exceptional mechanical efficiency, their realm extends from the primitive natural world to the modern sophisticated engineering, and many familiar materials are cellular, including plant stems, bones, feathers, foams, sponges, and sandwich panels, where strong, light-weight, energy-absorbing structures are desirable (Gibson et al., 2010; Gibson, 2005; Meyers et al., 2008). In some 'softer' biogenic or bio-inspired cellular materials, the deformation of the cell walls is entirely elastic (albeit large) and therefore recoverable, but at high strain, opposite cell walls may come into contact with each other and the material collapses by densification (or compaction), while the contacting walls continue to deform elastically as in a compact material (Weaire and Fortes, 1994). For example, the development of highly flexible stents and scaffolds for soft tissue re-growth in biomedical applications is a rapidly growing multidisciplinary area of biomaterials and tissue engineering, and many foams and sponges designed for cushioning and re-usability can also be found in everyday life as well as in several industrial areas, e.g. microelectronics, aerospace, pharmaceutical and food processes (Scanlon, 2005).

As for compact materials, it is possible to define an elastic limit for a cellular material: below this limit, the deformation is elastic and can be quite large, while at the elastic limit, the cells collapse by some form of mechanism, such as fracture, plastic deformation, or elastic buckling. Since the rich mechanical behaviour of cellular structures is due to the inextricable relation between the material and the geometry, general results explaining their mechanical

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behaviour cannot be expected (Bigoni, 2012). Nonetheless, important conclusions can be drawn in various special cases by applying suitable strategies.

At low stresses or strains, mathematical models for cellular solids which are based on the physical assumption that the cell walls are linearly elastic with a geometrically nonlinear behaviour are valid. In this context, a comprehensive study of micromechanical models of solid foams is provided by Gibson and Ashby (1997).

Currently, there is an increasing interest in the nonlinear response of periodic structures capable of large strain deformation, where nonlinear elasticity is expected to play a role. For example, in Michailides et al. (2009), multiscale stability aspects of the superelastic behaviour of hexagonal honeycombs under in-plane compression are analysed, and an in-depth parameter study is performed on the influence of different material laws on the finite strain deformation of honeycombs with perfect and imperfect geometries, while finite element simulations are shown to capture the behaviour observed in the experiments; in Siboni and Castaeda (2014), homogenisation estimates for the finite strain effective response of dielectric elastomer composites subject to electromechanical loading conditions are obtained from available estimates for the purely mechanical response combined with a partial decoupling approximation strategy; in Shan et al. (2014), a combination of experiments and finite element simulations of elastomeric porous structures show that, by controlling the loading direction, multiple pattern transformations can be induced by buckling, which can then be exploited to design tunable structural materials and devices. While particular structures seen in experiments can be mimicked computationally by various finite element implementations, there are still many interesting local phenomena that may occur in cellular structures made from nonlinear elastic materials, which are measurable though perhaps not visible, and are yet to be identified and exploited.

Our present goal is to *analyse and compute the deformation of seamless cellular bodies within the framework of finite strain elasticity*, which in principle can provide a complete description of elastic responses of the solid cell walls under loading, and show some interesting behaviours which are not captured in the small strain regime, and thus provide new insight into the rich mechanical response of these structures. In the field of large deformations, finite elasticity covers the simplest case where internal forces (stresses) only depend on the present deformation of the body and not on its history, so it excludes plasticity, viscosity, and damage (Antman, 2005; Green and Adkins, 1970; Ogden, 1997; Truesdell and Noll, 2004). Since damage by cell collapse involves additional conditions between the contacting cell walls, we assume that the loading conditions are such that the peak load is below the threshold that will lead to cell closure, and therefore contact between cell walls is not treated here.

Most cellular solids are anisotropic due to the structural distribution of the cells as well as the cell wall material aeolotropic properties. For example, at millimetre scale, plant stems are cellular solids, while at micrometre scale they are fibre reinforced composites. Here, we consider one of the most common features of many cellular solids, namely the orthotropic structural symmetry, whereby the structure has two or three orthogonal axes of symmetry. In our analysis, this is reflected first by the square geometry of the cells while the cell walls are assumed to be isotropic, then also by the orthotropic cell wall material for which the symmetry axes align with those arising from the structural geometry. The assumption of square shaped cells is then sufficient to prove that different local mechanical effects occur when the cell walls are made from a general nonlinear hyperelastic material than when the cell wall material is linearly elastic, which is the main scope of this paper. Interesting finite deformation effects caused by different cell shape geometries are analysed in Mihai and Goriely (2014).

For physical plausibility, we require that the cell wall material satisfies the Baker–Ericksen (BE) inequalities stating that *the greater principal stress occurs in the direction of the greater principal stretch* (Baker and Ericksen, 1954), and also the pressure-compression (PC) inequalities stating that *each principal stress is a pressure (compression) or a tension according as the corresponding principal stretch is a contraction or an elongation (extension)* (Truesdell and Noll, 2004, pp. 155–159). Since these inequalities are satisfied by most elastic materials, as confirmed by experiments and experience, our assumptions are not unduly restrictive. For linear elastic materials, if $\mu$ and $\kappa$ are the shear and the bulk modulus, respectively, then the PC inequalities take the form $\mu > 0$ and $\kappa > 0$, while BE inequalities are reduced to $\mu > 0$. However, in finite elasticity in general, neither of these two sets of inequalities is implied by the other.

In this context, for thin square and tubular cellular structures made from a nonlinear hyperelastic material, we show that, when these structures are subjected to uniform external conditions, such as stretch, shear, or torsion, internal non-uniform local deformations occur on the scale of the cell dimension. In many cellular structures, *such internal deformations may lead to further changes in the material properties as the deformation progresses*, as seen for example in Mihai and Goriely (2014), but their study is non-trivial since the corresponding stresses are non-trivial functions of position and material properties. Nonetheless, the insight gained from the analysis of the deformation for the underlying compact material can provide useful insight into the global behaviour of the structure. Accordingly, we begin our investigation with a uniform deformation, as in a compact material (Section 2), then set the cell walls free by removing the traction constraints at the pre-deformed walls, while the external conditions are maintained (Sections 3–6). *This is equivalent to decomposing the large deformation of the cellular structure into two successive deformations: (i) one where a uniform deformation occurs everywhere, as in a compact solid, and (ii) one where the traction constraints at the pre-deformed cell walls are removed, so that the walls deform freely.* By employing this successive decomposition procedure (SDP), we exploit the fact that, under large loading conditions, the deformation of the compact material is potentially closer to that of the cellular structure than the undeformed state, and can be easier to predict, making the final configuration of the structure more tractable. Specifically, for the elastic structures analysed here, an *isochoric deformation* that can be maintained in both compressible and incompressible materials is considered at the first step; then the stresses in this known configuration were used to analyse the *free shape problem* at the second step, where the geometry of the individual cell walls also plays a role.

In this sense, the proposed SDP provides a general mathematical setting applicable to a class of cellular structures made from different hyperelastic materials or containing different numbers of cells, and for which analogous results can be predicted that demonstrate some fundamental differences between the local behaviour of cellular bodies with nonlinear elastic cell walls and those made from linearly elastic materials.

For numerical illustration of the nonlinear mechanical effects under investigation, in Sections 3.1,4.1,5.1 and 6.1, finite element simulations of representative model structures with a small number of cells made from a Mooney material are presented, while in Section 7, the effects of increased cell wall thickness in some chosen regions and reinforcing fibres are also considered. For these computer simulations, the size of the cell and the size of the structure are comparable, and therefore the nonlinear mechanical effects at the cell level are visible directly at the structural level. These model simulations were produced using the standard finite element procedure available within then open-source software Finite Elements for Biomechanics (FEBio) environment (Maas et al., 2012), and are intended as a supplement to show how local nonlinear effects