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# Peeling of an elastic membrane tape adhered to a substrate by a uniform cohesive traction

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ABSTRACT

An analytical model is provided for the peeling of a tape from a surface to which it adheres through cohesive tractions. The tape is considered to be a membrane without bending stiffness and is initially attached everywhere to a flat rigid surface. The tape is assumed to deform in plane strain, and finite deformations in the form of elastic strains are accounted for. The cohesive tractions are taken to be uniform when the tape is within a critical interaction distance from the substrate and then to fall immediately to zero once this critical interaction distance is exceeded. When the distance between the tape and the substrate is zero, repulsive and attractive tractions balance to zero; in this segment, sliding of the tape relative to the substrate is forbidden when we pull the tape up somewhere in the middle, though we permit such sliding when the tape is peeled from one end. In the cohesive zone and where the tape is detached, the interaction of the tape with the substrate is frictionless. Results are given for the force to peel a neo-Hookean tape at any angle up to vertical when one end of it is pulled away from the substrate, as well as for scenarios when the tape is lifted somewhere in the middle to form a V shape being pulled away from the substrate.

## 1. Introduction

Detachment of thin, flexible films by peeling is a ubiquitous phenomenon of practical importance to a wide range of problems. Examples include the reliability of coatings (Bainbridge et al., 1982; Ghosh et al., 1998; Kurzweg et al., 1998), adhesive tapes used to fix objects in place (Gent and Kaang, 1986; Williams and Kauzlarich, 2005; Sun et al., 2013), the transfer of graphene sheets from one surface to another (Lu and Dunn, 2010), and the ability of plants and animals to cling to surfaces *e.g.*, ivy (Melzer et al., 2010), geckos (Pesika et al., 2007; Cheng et al., 2012; Sauer, 2011) and mussels (Waite et al., 2005).

The motivation for the present study is provided by the latter applications, which involve the peeling of compliant, elastic films and relatively weak interface bonding (*e.g.*, van der Waals forces), which allows for sliding in the attached region of the film. As is well known, weak bonding does not necessarily imply low detachment forces, since the latter can be strongly influenced by dissipative processes in the detachment process zone (*e.g.*, friction). Moreover, these applications often involve peeling from a fully attached state wherein the entire film is adhered (as opposed to the application of force to an already detached end of the film).

Though peeling has been extensively studied (Gent and Kaang, 1986; Williams and Kauzlarich, 2005; Pesika et al., 2007; Cheng et al., 2012; Sauer, 2011; Kendall, 1971,1975; Kim and Aravas, 1988; Kim and Kim, 1988; Kim et al., 1989; Wei and Hutchinson, 1998; Rahulkumar et al., 2000; Yang et al., 2000; Georgiou et al., 2003; Plaut and Ritchie, 2004; Thouless and Yang, 2008; Thouless and Jensen, 1992; Begley et al., 2013; Kinloch et al., 1994; Kroner et al., 2011; Williams and Hadavinia, 2002; Wan and Julien, 2009; Molinari and Ravichandran, 2008), the system properties described above require a combination of behaviors not previously considered: the analysis must account for large elastic deformations, the possibility of sliding in the attached region prior to detachment, and the possibility of detachment from a fully adhered state (as opposed to a tape that already has a detached segment at one end). The first of these two behaviors

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**Nomenclature**

Latin

| | |
|---|---|
| $B$ | point on the tape at the open end of the cohesive zone before peeling commenced |
| $E$ | Young's modulus of the tape |
| $F$ | applied force |
| $G$ | energy released per unit area of tape peeled |
| $L$ | position where the apex of the $V$ was originally attached |
| $N$ | axial tension in the tape |
| $T$ | cohesive tractions (that depend on the distance between the tape and the substrate) |
| $X$ | position in the tape before it is lifted off the substrate |
| $b$ | current position of the open end of cohesive zone |
| $h$ | cross-sectional height dimension of the unstressed tape |
| $s$ | arc length (along the deformed tape) |
| $t$ | 1st Piola–Kirchhoff stress |
| $w$ | cross-sectional width dimension of the unstressed tape |
| $x$ | deformed position of the tape on the abscissa |
| $y$ | distance between the deformed tape and the substrate |

Greek

| | |
|---|---|
| $\Gamma$ | adhesion energy per unit area of the undeformed tape |
| $\Delta$ | distance from the substrate to the apex of the $V$ |
| $\Lambda$ | axial stretch ratio for the attached tape |
| $\Sigma$ | nominal cohesive traction, giving a force per unit area of the undeformed tape |
| $\delta$ | critical interaction distance |
| $\theta$ | angle between the tape and a line parallel to the substrate surface |
| $\lambda$ | axial stretch ratio of the deformed tape |
| $\xi$ | variable of integration |
| $\sigma_H$ | Lagrange multiplier |
| $\psi$ | energy per unit volume stored in the tape due to deformation |

Subscripts

| | |
|---|---|
| $A$ | attached tape segment |
| $D$ | completely detached tape segment |
| $e$ | end of the tape |
| $i = 1, 2, 3$ | principal axes |
| $o$ | original position |

has been previously modeled assuming negligible bending stiffness (Begley et al., 2013), and produces very different predictions for detachment forces than classical models of peeling that assume pure sticking at the interface (Kendall, 1971, 1975).

In this previous model for detachment with sliding (Begley et al., 2013), the work done in frictional sliding is explicitly modeled and the adhesion energy controlling detachment is the purely normal work of separation. The model predicts that the force required for peeling rises without limit as the angle between the applied force and the substrate decreases, a consequence of the fact that lateral sliding prior to detachment is permitted and yet is not factored into the energy released by peeling.

An alternative approach, exemplified in mixed-mode delamination models (Thouless and Yang, 2008; Thouless and Jensen, 1992; Hutchinson and Suo, 1992; Li et al., 2004), is to empirically account for the dissipated energy in the detachment process zone by invoking an adhesion energy that depends on the relative amounts of sliding and normal separation in the process zone (or mode-mixity). The two approaches can be brought into coincidence by defining a mixed-mode adhesion energy such that the peel force predicted via pure sticking (Kendall, 1971, 1975) is equivalent to the sliding model. Put another way, the sliding model (Begley et al., 2013) creates the opportunity to predict mode-dependent adhesion energy; this exercise yields a predicted mixed-mode adhesion energy that is quite similar to empirical forms typically adopted for use with pure sticking peeling models.

Since the previous treatment of peeling with sliding utilizes adhesion energy as a single parameter controlling detachment, it cannot be used to predict detachment of a fully adhered film. Detachment in this scenario is triggered by displacements reaching the critical value required for separation. Hence, predicting detachment of a fully adhered film requires explicit reference to the traction-displacement cohesive law controlling adhesion. In this work, we assume a Dugdale-type (Dugdale, 1960) cohesive law for normal separations, and, in the single-sided peel case, explicitly allow for sliding with a constant sliding stress. In contrast to previous cohesive models, we do not assume that detachment can be driven by lateral sliding. Thus, the current analysis is unique in the following respects: (i) it allows for large *elastic* deformation, (ii) it assumes detachment occurs only by normal separation, (iii) in the single-sided peel case it allows for frictional sliding in the attached region of the film, and (iv) it invokes a two parameter cohesive law, as is required to predict detachment from a fully-adhered state.

While this combination of features has not been previously considered, in one form or another the individual features have been included in prior treatments (Kim and Aravas, 1988; Kim and Kim, 1988; Kim et al., 1989; Wei and Hutchinson, 1998; Rahulkumar et al., 2000; Yang et al., 2000; Georgiou et al., 2003; Plaut and Ritchie, 2004; Thouless and Yang, 2008; Thouless and Jensen, 1992; Begley et al., 2013; Kinloch et al., 1994; Kroner et al., 2011; Williams and Hadavinia, 2002; Wan and Julien, 2009; Molinari and Ravichandran, 2008) and provide significant insight regarding the mechanics of peeling. This work includes treatments of large elastic–plastic deformations (Kim and Aravas, 1988; Kim and Kim, 1988; Kim et al., 1989; Wei and Hutchinson, 1998; Rahulkumar et al., 2000; Yang et al., 2000; Kinloch et al., 1994), mode-mixity effects for interfaces with pure sticking (Thouless and Yang, 2008; Thouless and Jensen, 1992), the impact of cohesive properties on peeling from a detached end (Rahulkumar et al., 2000; Georgiou et al., 2003; Plaut and Ritchie, 2004; Williams and Hadavinia, 2002; Wan and Julien, 2009), and large deformation for sticking interfaces (Molinari and Ravichandran, 2008). Importantly, this prior work enables one to draw comparisons between the present approach and mixed-mode delamination frameworks. Previous treatments of peeling that explore the relative importance of bending and stretching during peeling are particularly noteworthy; in the present approach, bending deformation is neglected and the film is treated as a membrane. This approximation is motivated here by the fact that it enables closed-form solutions that yield general insight, and the fact that it is a clearly valid limit for thin, compliant films. That said, it is worth emphasizing that more sophisticated treatments that address the influence of bending at the edge of attachment are available (Sauer, 2011; Thouless and Yang, 2008), and can be used to identify limits in which a membrane approximation is likely to be valid.

## 2. Overview of the model

Consider a tape completely stuck to a flat, rigid substrate, as shown in Fig. 1(a). The tape is very thin, and may have been