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Magnetoactive elastomers with periodic and random microstructures

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ABSTRACT

We investigate the behavior of magnetoactive elastomers (MAEs) with periodic and random distributions of circular and elliptical fibers. For the MAEs with periodic microstructures, we develop finite element models and determine the local fields as well as the effective properties of MAEs with rectangular and quasi-hexagonal unit cells. For the MAEs with random microstructures, we derive a closed-form expression for the effective response making use of a recently developed theory (Ponte Castañeda and Galipeau, 2011). In particular, we determine the responses to pure shear loading in the presence of a magnetic field, both of which are aligned with the geometric axes of the fibers, and examine the roles of the deformation, concentration, particle shape, and distribution on the magnetostriction, actuation stress, and the magnetically induced stiffness of the composite. We show that the coupling effects are of second order in the concentration. This is consistent with the fact that these effects are primarily the result of the interaction between inclusions. We also demonstrate explicitly that the magnetomechanical coupling of these MAEs, when subjected to aligned loading conditions, depends not only on the magnetic susceptibility, but also, crucially, on its derivative with respect to the deformation. As a consequence, we find that the magneto-elastic effects may be quite different, even for composites with similar effective susceptibilities.

1. Introduction

Magnetoactive elastomers (MAEs) are composite materials exhibiting coupled magnetic and mechanical behavior. In this work we examine typical MAEs consisting of magnetically susceptible particles embedded in a non-magnetic soft elastomer matrix. Frequently used magnetic materials include carbonyl iron and nickel; examples of more exotic inclusions are Terfenol-D and Ni_2MnGa. MAEs are of interest because magnetic fields are capable of modifying the effective stiffness of the composite and of producing magnetostrictive strains. Both effects take place quickly and reversibly, making MAEs good candidates for tunable vibration dampers and magnetic actuators.

For MAEs made with inclusion materials such as carbonyl iron, nickel, or cobalt, which are effectively rigid compared to the elastomer matrix, the principal mechanisms are magnetic torques and magnetic interactions between particles (Jolly et al., 1996; Bednarek, 1999; Ginder et al., 2002; Guan et al., 2008). For the particular case when the magnetic particles are aligned with the external magnetic field, there are no magnetic torques on the particles and the magnetoelastic effects are controlled by particle interactions. Various approaches have been used to directly account for particle pair forces in MAEs in the context of infinitisemal deformations, including the works of Borcea and Bruno (2001), Yin and Sun (2006), and Yin et al. (2006).

Magnetic interactions in deformable elastic media can also be accounted for, in the context of a thermodynamically consistent formulation, by means of a free-energy function, leading to the notion of magnetic stresses, which exist even in vacuum (Maxwell, 1873). The pioneering works on electro- and magneto-elastic behavior of a continuum by Toupin (1956), Truesdell and Toupin (1960), Tiersten (1964), Brown (1966), and Maugin and Eringen (1972) have been recently reviewed and further developed by various authors (Brigadnov and Dorfmann, 2003; Dorfmann and Ogden, 2004a,b; Kankanala and Triantafyllidis, 2004; Vu and Steinmann, 2007; Bustamante et al., 2008). Making use of these constitutive formulations, Ponte Castañeda and Galipeau (2011) proposed a finite-strain, variational homogenization framework to determine the total magneto-elastic stress arising in a composite material as a consequence of combined magnetic and mechanical stimuli. Furthermore, for the special case of MAEs, where the magnetic particles are rigid compared to the soft elastomer matrix, Ponte Castañeda and Galipeau (2011) showed that the total stress

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can be expressed as the sum of the purely mechanical stress, which exists in the composite when no magnetic fields are applied, together with the Maxwell stress in vacuum and an extra magnetic stress which is determined by the derivative of the (deformation-dependent) magnetic susceptibility of the composite with respect to the stretch. Applications of these results for MAEs with random microstructures and for magnetic fields that are aligned and unaligned with the anisotropic axes have been given by Galipeau and Ponte Castañeda (2012) and Galipeau and Ponte Castañeda (2013a), respectively. The mathematically analogous case of dielectric elastomer composites with periodic and random microstructures was considered by Ponte Castañeda and Siboni (2012) and Siboni and Ponte Castañeda (2013), respectively. In addition, deBotton et al. (2007) computed directly the macroscopic constitutive relations for electroelastic composites with layered microstructures, taking advantage of the fact that the fields are uniform in the layers. The stability of electroactive laminates was considered by Rudykh and deBotton (2011). The ability to significantly enhance the electromechanical coupling with appropriate arrangement of the microstructure of the composite was demonstrated in Tian et al. (2012) and Rudykh et al. (2013). Also, Galipeau and Ponte Castañeda (2013b) have recently shown that giant magnetostriction can be achieved in composites with herringbone-type microstructures by combining the action of magnetic torques with soft mechanical modes of deformation in the elastomer phase. Shear localization instabilities in layered and particulate magneto-elastic composites have been considered recently by Rudykh and Bertoldi (2013) and Galipeau and Ponte Castañeda (2013a), respectively. More general instabilities in the context of layered dielectric elastomers have been considered by Bertoldi and Gei (2011) and Rudykh et al. (2014).

Unfortunately, the set of microstructures for which exact analytical solutions can be obtained is essentially limited to materials with layered microstructures. For magneto-elastic materials with more general periodic microstructures, which have great potential for enhancing magneto-elastic performance, the exact behavior may be obtained by numerical computations. The finite element (FE) method is usually employed (Rudykh and deBotton, 2012) for this purpose. In this work, we pursue this approach for the coupled magnetomechanical problem and construct FE models for solving the magnetomechanical problem under finite deformations and periodic boundary conditions. Specifically, we examine periodic MAEs with (i) *rectangular* and (ii) *quasi-hexagonal* periodicity. The FE models provide the information about the local fields, which can be averaged over the unit cell to obtain the effective properties of the composites. For random microstructures, we estimate the effective behavior of MAEs with the homogenization-based constitutive model recently developed in Galipeau and Ponte Castañeda (2013a).

We define key parameters that govern the coupled magnetomechanical behavior of MAEs. These parameters are directly related to the applied traction measured on the surface of the material while accounting for the magnetic stresses outside the material. The governing parameters of the magnetomechanical coupling are evaluated for MAEs with random, quasi-hexagonal, and rectangular periodic microstructures over a wide range of concentrations and particle aspect ratios. We demonstrate explicitly that the magnetomechanical coupling depends not only on the magnetic susceptibility, but, more importantly, also on its derivative with respect to deformation. Accordingly, it is demonstrated that linearly magnetic materials with similar susceptibilities can exhibit rather different magneto-elastic coupling. Moreover, we find that, for some composites, while the magnetic induced tractions are larger, the magnetostriction is lower and vice versa. The two competing mechanisms that are responsible for this complex behavior are identified and discussed. Finally, in order to shed light on the complex dependence of the magneto-elastic coupling on the microstructure of the composite, we provide a qualitative analysis of this relation in terms of the magnetomechanical interactions among the inclusions.

In this work scalars will be denoted by italic Roman, a and G, or Greek letters, α and Γ; vectors by boldface Roman letters, $\mathbf{b}$; second-order tensors by boldface italic Roman letters, $\boldsymbol{P}$, or bold face Greek letters, $\boldsymbol{\epsilon}$. When necessary Cartesian components will be used; for example, P_{ij} are the Cartesian components of $\boldsymbol{P}$.

2. Magneto-elasticity in the quasistatic regime

Consider the quasistatic deformation of a body. In its reference configuration, the location of each material point is defined by the position vector $\mathbf{X}$. Under the combined action of mechanical and magnetic effects, the body deforms. In the deformed configuration, the new position of the material points is described by $\mathbf{x}$. The local deformation is characterized by the deformation gradient $\boldsymbol{F} = \text{Grad}\,\mathbf{x}$, with Cartesian components $F_{ij} = \frac{\partial x_i}{\partial X_j}$, and is such that $J = \det \boldsymbol{F} > 0$. Conservation of mass implies that locally $\rho_0 = \rho J$, where ρ_0 and ρ are the material densities in the reference and deformed configurations, respectively. We also recall that the polar decomposition of the deformation gradient is $\boldsymbol{F} = \boldsymbol{R}\boldsymbol{U}$, where $\boldsymbol{R}$ is the rotation and $\boldsymbol{U}$ is the stretch tensor.

We define $\boldsymbol{T}$ and $\boldsymbol{S} = J\boldsymbol{T}\boldsymbol{F}^{-T}$ to be the *total* Cauchy and (first) Piola–Kirchhoff stress tensors, respectively, which at static equilibrium and in the absence of body forces satisfy the equivalent mechanical equilibrium conditions

$$\text{div}\,\boldsymbol{T} = 0 \quad \text{or} \quad \text{Div}\,\boldsymbol{S} = 0. \tag{1}$$

The operators div and Div are the divergence operators with respect to $\mathbf{x}$ and $\mathbf{X}$, respectively. Together with the linear momentum balance Eq. (1), the stress fields also satisfy the balance of angular momentum. Accordingly, $\boldsymbol{T}^{\mathrm{T}} = \boldsymbol{T}$, or equivalently, $\boldsymbol{S}\boldsymbol{F}^{T} = \boldsymbol{F}\boldsymbol{S}^{T}$. The stress may be discontinuous across an interface, but must satisfy the jump conditions

$$[[\boldsymbol{T}]]\mathbf{n} = 0 \quad \text{or} \quad [[\boldsymbol{S}]]\mathbf{N} = 0, \tag{2}$$

where $\mathbf{n}$ and $\mathbf{N}$ denote the normal to the interface in the deformed and reference configurations, respectively.

The magnetic field is characterized by two primary magnetic field vectors: the magnetic induction $\mathbf{b}$ and the magnetic intensity $\mathbf{h}$, both defined in the current configuration. In the absence of surface charge and free currents, and for quasi-static conditions, they satisfy the field equations

$$\text{div}\,\mathbf{b} = 0 \quad \text{and} \quad \text{curl}\,\mathbf{h} = 0, \tag{3}$$

where the curl operator is with respect to $\mathbf{x}$. Alternatively, following the work of Dorfmann and Ogden (2004a), these equations can be written in Lagrangian form as

$$\text{Div}\,\mathbf{B} = 0 \quad \text{and} \quad \text{Curl}\,\mathbf{H} = 0, \tag{4}$$

where $\mathbf{B} = J\boldsymbol{F}^{-1}\mathbf{b}$ and $\mathbf{H} = \boldsymbol{F}^{T}\mathbf{h}$ are the Lagrangian counterparts of the magnetic fields and the Div and Curl operators are with respect to $\mathbf{X}$. The corresponding jump conditions at an interface are

$$[[\mathbf{b}]] \cdot \mathbf{n} = 0 \quad \text{and} \quad [[\mathbf{h}]] \times \mathbf{n} = 0, \tag{5}$$

or

$$[[\mathbf{B}]] \cdot \mathbf{N} = 0 \quad \text{and} \quad [[\mathbf{H}]] \times \mathbf{N} = 0. \tag{6}$$

The relation between the magnetic fields is customarily defined in terms of the magnetization $\mathbf{m}$, such that

$$\mathbf{h} = \frac{1}{\mu_0}\mathbf{b} - \mathbf{m}, \tag{7}$$