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Elastic-viscoplastic notch correction methods

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ABSTRACT

Neuber's type methods are dedicated to obtain fast estimation of elastic–plastic state at stress concentrations from elastic results. To deal with complex loadings, empirical rules are necessary and do not always give satisfying results. In this context, we propose a new approach based on homogenization techniques. The plastic zone is viewed as an inclusion in an infinite elastic matrix which results in relationships between the elastic solution of the problem and estimated stress–strain state at the notch tip. Three versions of the notch correction method are successively introduced, a linear one which directly uses Eshelby's solution to compute stresses and strains at the notch, a non-linear method that takes into account plastic accommodation through a β-rule correction and, finally, the extended method that is based on the transformation field analysis methods. All the notch correction methods need calibration of localization tensors. The corresponding procedures are proposed and analyzed. The methods are compared on different simulation cases of notched specimens and the predictive capabilities of the extended method in situations where plasticity is not confined at the notch are demonstrated. Finally, the case of a complex multiperforated specimen is addressed.

1. Introduction

Lifetime of structural components is often controlled by notches and stress concentrations where plasticity can develop. Finite element (FE) elastic–plastic or elastic-viscoplastic simulations of complex components can still be prohibitive in a design process. Consequently, there is a need for fast estimation methods of plasticity at stress concentrations.

Rules applying a plastic correction to deduce elastic–plastic stress and strain state from an elastic solution were developed to do so. Neuber (1961) was the first to propose such a method for uniaxial monotonic loading conditions. In case of notched bodies in plane stress, which results in a uniaxial stress state, he postulates a kind of local energetic equivalence between an elastic and an elastic–plastic calculation:

$$\sigma^e_{22}\epsilon^e_{22} = \sigma^p_{22}\epsilon^p_{22} \quad (1)$$

Later, Molski and Glinka (1981) developed a similar method assuming localized plasticity at the notch tip. In that case, the strain energy density at the notch tip can be approximated by that obtained if the body were to remain elastic:

$$\int_0^{\varepsilon^a} \sigma^p_{22} d\varepsilon^p_{22} = \frac{1}{2}\sigma^e_{22}\varepsilon^e_{22} \quad (2)$$

Theoretical justifications of those approaches have been proposed in the literature (Desmorat, 2002; Ye et al., 2004; Guo et al., 1998). They were also extended and improved over the last four decades. For example, Chaudonneret and Culié (1985) have worked on cyclic extensions.

One of the main issues with those methods arises when dealing with multiaxial stress states. In the general case of triaxial mechanical state at the notch root, three stress components, four strain components and four plastic strain components have to be computed locally. Elastic and plastic behavior laws provide 4 + 4 scalar equations. Consequently, three more equations are needed to solve the problem.

In the one hand, the approach followed in Hoffman and Seeger (1985) consists in generalizing the uniaxial Neuber rule using equivalent stress and strain quantities instead of uniaxial values. They add two more assumptions on (i) principal directions and (ii) ratios between the two first principal stresses to close the problem. In a similar way, Moftakhar et al. (1994) proposed multiaxial generalizations of Neuber's and Molski–Glinka's rules and assumes equality of the contribution of each stress–strain component in the

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strain energy density between elastic and elastic–plastic computations.

On the other hand, researchers have worked on notch correction rules for non-proportional multiaxial loadings. Barkey et al. (1994) and Köttgen et al. (1995) incorporate directly the notch influence into the constitutive equation. An anisotropic structural yield surface in nominal stress space is then introduced. Another way to treat non-proportional loading sequences consists in applying the incremental formulation of generalized Neuber's rule as in Buczynski et al. (2003).

However, all the above presented methods suffer some limitations: (i) they are often limited to given geometries and (ii) they cannot take plastic redistributions into account. More recently Herbland et al. (submitted for publication) has proposed a completely new approach based on the Eshelby inclusion theory. The notch tip is viewed as an inclusion in an infinite matrix. The general formulation of this method allows the possibility to address non-proportional loading sequences for any material model. Herbland have also proposed a non-linear extension to take plastic redistributions around notch tip into account. Indeed, large plastic zones at notch tips are still challenging issues and most of Neuber's type methods fail in predicting plastic accomodation and ratcheting phenomenon.

The first objective of the present contribution is thus to discuss the predicting capabilities of Herbland's methods in non-confined plastic zone cases. This article aims also at presenting a new robust correction method that extends Herbland's work. It follows the same idea but it is derived from the transformation field analysis (TFA) (Dvorak and Benveniste, 1992) approach developed in the homogenization literature.

This contribution is organized as follows. Section 2 of this paper briefly sums up Herbland's linear and non-linear correction methods. The tested geometries and material models are presented in Section 3. Section 4 is dedicated to the application of both linear and non-linear Herbland's methods. The new correction method we propose is presented together with its application in Section 5. Finally, both Herbland's and the new correction method have been validated on a multiperforated specimen as described in Section 6.

2. Linear and non-linear notch correction methods

In Herbland's method (Herbland et al., submitted for publication), the plastic zone at notch tip is seen as an inclusion in a semi-infinite matrix. In the case of an infinite elastic matrix, Eshelby's solution links the stress in the inclusion $\underset{\sim}{\boldsymbol{\sigma}}^I$ and in the matrix $\underset{\sim}{\boldsymbol{\sigma}}^M$:

$$\underset{\sim}{\boldsymbol{\sigma}}^I = \underset{\sim}{\boldsymbol{\sigma}}^M + \underset{\approx}{\boldsymbol{C}} : \left(\underset{\sim}{\boldsymbol{\varepsilon}}^{pM} - \underset{\sim}{\boldsymbol{\varepsilon}}^{pI}\right) \quad (3)$$

where the superscript M denotes matrix quantities and the superscript I denotes inclusion quantities. The fourth order tensor $\underset{\approx}{\boldsymbol{C}}$ depends on the elastic properties of the material and the geometry of the inclusion. This type of stress redistribution is the basis of Kröner's model for polycrystals. It is valid at the onset of plastic flow. If the problem is restricted to confined plasticity at the notch tip, the plastic deformation in the matrix $\underset{\sim}{\boldsymbol{\varepsilon}}^{pM}$ is supposed to be equal to zero so that Eq. (3) reduces to:

$$\underset{\sim}{\boldsymbol{\sigma}}^I = \underset{\sim}{\boldsymbol{\sigma}}^M - \underset{\approx}{\boldsymbol{C}} : \underset{\sim}{\boldsymbol{\varepsilon}}^{pI} \quad (4)$$

In the notch correction framework and following Neuber's type of approaches, the superscript M stands for the quantities at the notch tip coming from the elastic computation and the superscript I denotes the quantities in the elastic–plastic case. Nevertheless, other definitions for $\underset{\sim}{\boldsymbol{\sigma}}^M$ are discussed in Herbland et al. (submitted for publication) (nominal stress, average over a volume around the notch tip). This method will be denoted CL in the following. In fact, the analogy with the homogenization models is not fully verified, since, due to the introduction of the free surface, the stress state is not uniform in the plastic zone, and some stress components are null.

As classically shown in the homogenization framework (Berveiller and Zaoui, 1978), this linear correction leads to elastic accomodation. Herbland proposed an extension of his method to take into account plastic accomodation, still using tools from the homogenization literature. He applied the β-rule (Cailletaud, 1987) which consists in replacing the plastic strain $\underset{\sim}{\boldsymbol{\varepsilon}}^{pI}$ by an auxiliary variable $\underset{\sim}{\boldsymbol{\beta}}^I$ whose evolution is governed by a non-linear equation. More precisely, Herbland proposed the following evolution equation for $\underset{\sim}{\boldsymbol{\beta}}^I$ to control ratcheting effect:

$$\dot{\underset{\sim}{\boldsymbol{\beta}}}^I = \dot{\underset{\sim}{\boldsymbol{\varepsilon}}}^{pI} - \underset{\approx}{\boldsymbol{D}} : \left(\underset{\sim}{\boldsymbol{\beta}}^I - \underset{\approx}{\boldsymbol{\delta}} : \underset{\sim}{\boldsymbol{\varepsilon}}^{pI}\right) \| \dot{\underset{\sim}{\boldsymbol{\varepsilon}}}^{pI} \| \quad (5)$$

The tensor $\underset{\approx}{\boldsymbol{\delta}}$ is introduced to limit the ratcheting effect due to the non-linear term. It is diagonal and writes, using Voigt notations:

$$\underset{\approx}{\boldsymbol{\delta}} = \begin{pmatrix} \delta & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta/2 \end{pmatrix} \quad (6)$$

Herbland's non-linear correction method will be denoted CNL in the following.

The linear and non-linear methods are summed up by Fig. 1. In a first step, the tensor $\underset{\approx}{\boldsymbol{C}}$ (plus $\underset{\approx}{\boldsymbol{D}}$, δ for the non-linear method) are calibrated through FE simulations. As it will be explained later, assumptions on the shape of those tensors can be made to reduce the number of parameters to be identified. First, an elastic-visco-plastic FE simulation on a monotonic or few cycles (typically 5 cycles) will be used as a reference. Then, an elastic simulation is performed to get $\underset{\sim}{\boldsymbol{\sigma}}^M$ at notch tip. This elastic simulation is post-processed to estimate the stress and strain fields at notch tip using Eq. (4) and the constitutive equations of the material. Reference values and notch correction values for $\underset{\sim}{\boldsymbol{\sigma}}$ and $\underset{\sim}{\boldsymbol{\varepsilon}}$ are then compared in an optimization loop to obtain the optimal values for the method parameters $\underset{\approx}{\boldsymbol{C}}$, $\underset{\approx}{\boldsymbol{D}}$, δ. In a second step, the notch correction method can be used on the same geometry and for the same

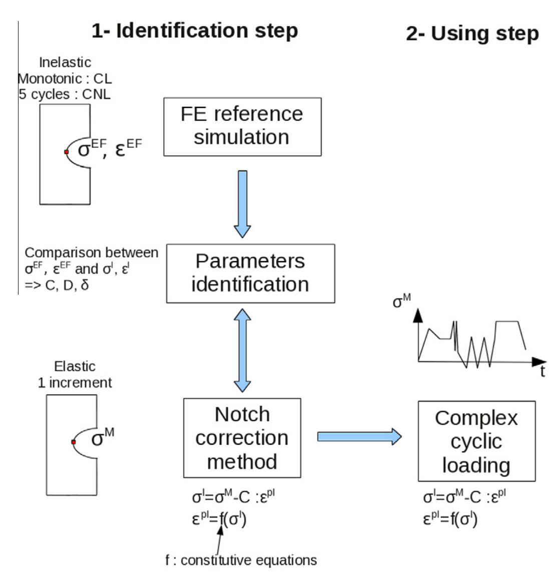

Fig. 1. Schematic view of the linear and non linear correction methods.