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Deformation and vibration of upright loops on a foundation and of hanging loops

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ABSTRACT

The deformation and vibration of vertical flexible loops are investigated theoretically and experimentally. Both upright and hanging loops are considered. Potential applications include nanorings and carbon nanotubes as force sensors or structural components. The upright tubes rest on a rigid or linearly elastic (Winkler) foundation, and cases with adhesion and nonlocal elasticity are included in the analysis. The hanging loops are suspended by a clamp with zero or finite length. The effects of self-weight, foundation stiffness, work of adhesion, and nonlocal elasticity on the loop height or depth are determined, as well as the effects of self-weight and foundation stiffness on the lowest frequency for in-plane symmetric vibration. Good agreement is attained between theoretical results (based on an inextensible-elastica model) and experimental data.

1. Introduction

This paper treats a narrow strip formed into a vertical circular loop that is either upright (resting on a foundation) or hangs downward (suspended by a zero-length or finite-length clamp), subjected to its self-weight. Such a strip is sometimes called a ribbon. The problem can be related to cylindrical shells, nanorings, and carbon nanotubes. Equilibrium states are investigated, and in some cases small in-plane symmetric vibration about those configurations is considered. An analysis based on an inextensible elastica is carried out, and experiments are conducted. In the "basic problem", the loop is upright and the foundation is rigid. Extensions that are examined for upright loops include a linearly elastic (Winkler) foundation, adhesion between the loop and the foundation, and nonlocal elasticity of the loop.

The basic problem was analyzed by Wang and Watson (1981). Typical equilibrium shapes are depicted in Fig. 1. Similar shapes were obtained in Raux et al. (2010), theoretically and experimentally, for ribbons made of an elastic polymer. Hertel et al. (1998) and Pantano et al. (2004) showed such cross-sectional shapes for horizontal carbon nanotubes on a graphite substrate, with self-weight neglected and with deformations caused by van der Waals forces between the nanotube and the substrate. A similar problem was examined in Liu and Xia (2013), where the ends of a carbon nanotube were bent and attached to form a nanoring that was placed vertically on a horizontal substrate. The nanoring was modeled as an inextensible elastica.

Zheng and Ke (2010, 2011) considered a nanoring comprised of a bent carbon nanotube or a bundle of them. The ring was placed vertically on a horizontal substrate, and again self-weight was neglected and adhesion between the structure and the substrate was included. A vertical force (downward or upward) was applied at the top, and the authors referred to possible applications as a force sensor or a structural component in a nanoscale system. An inextensible-elastica analysis was performed, along with experiments. In the analysis, repulsive van der Waals forces induced a small separation between the nanoring and the substrate.

Finally, Shi et al. (2012, 2013) analyzed cross-sectional deformations of a small cylindrical shell resting on a horizontal substrate and subjected to a vertical force (downward or upward) at the top. Self-weight was neglected, and internal pressure was included (applicable to liposomes and biological cells). The attractive-repulsive interaction between the nanotube and the substrate was modeled by a point-to-point force acting perpendicularly to the cross section or to the substrate, rather than integrating a distributed force over the adjacent surface (Plaut et al., 2012).

The basic problem is formulated in Section 2, including the hanging loop, and the experiments are described in Section 3. Equilibrium results for the basic problem are shown in Section 4, and

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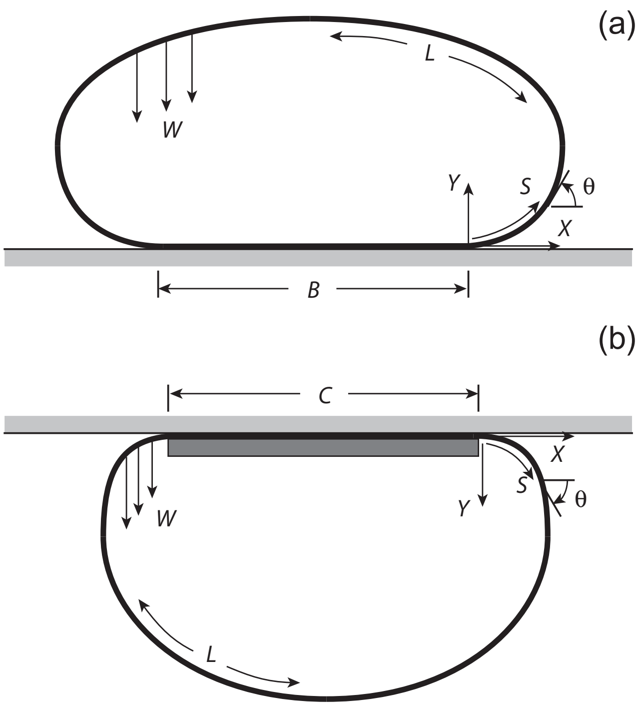

Fig. 1. Geometry of loop subjected to self-weight: (a) upright on rigid foundation; (b) hanging.

extended to include a Winkler foundation in Section 5, adhesion in Section 6, and nonlocal elasticity with adhesion in Section 7. Vibrations are treated in Section 8, and concluding remarks are given in Section 9. All results are presented in nondimensional form.

2. Formulation of basic problem

Four cases were delineated in Wang and Watson (1981) for upright loops. In Case I, for sufficiently low self-weight, the loop contacts the substrate only at the lowest point of the loop. In Case II, for a higher range of self-weight, a flat segment of the bottom of the loop is in contact with the substrate (see Fig. 1). In Case III, for a still higher range of self-weight, the bottom of the loop is in contact over a flat segment and the point that started at the top of the loop (for small self-weight) is in contact with the bottom of the loop. Finally, in Case IV, for high self-weight, a segment of the previous top part of the loop is flat on top of the bottom flat segment, with raised sections past the left and right ends of that segment.

First consider the upright loop, with Case II sketched in Fig. 1(a). It has circumference L, contact length B with the horizontal rigid foundation, weight W per unit length, modulus of elasticity E, width B_0, thickness H, cross-sectional area $A = B_0H$, and moment of inertia $I = B_0H^3/12$. With the origin at the right lift-off point, the arc length is S, the horizontal coordinate is $X(S,T)$, the vertical coordinate is $Y(S,T)$, and the angle between the horizontal and the tangent to the loop is $\theta(S,T)$, where T denotes time. The internal forces are $P(S,T)$ and $Q(S,T)$ parallel to the X and Y axes, respectively, and the bending moment is $M(S,T)$. On a positive face, P is positive if in the $-X$ direction, Q is positive if in the $-Y$ direction, and M is positive if counter-clockwise.

The loop is modeled as an inextensible elastica that is unstrained when straight. The governing equations for $0 < S < L - B$ for Cases I–III, based on geometry, moment–curvature relation, and equilibrium of an element including inertia forces, are

$$\begin{aligned}
&\frac{\partial X}{\partial S} = \cos\theta, \quad \frac{\partial Y}{\partial S} = \sin\theta, \\
&EI\frac{\partial \theta}{\partial S} = M, \quad \frac{\partial M}{\partial S} = Q\cos\theta - P\sin\theta, \\
&\frac{\partial P}{\partial S} = -(W/g)\frac{\partial^2 X}{\partial T^2}, \quad \frac{\partial Q}{\partial S} = -W - (W/g)\frac{\partial^2 Y}{\partial T^2}
\end{aligned} \tag{1}$$

(Santillan et al., 2006).

The analysis is conducted in terms of the following nondimensional quantities:

$$\begin{aligned}
&w = WL^3/EI, \quad x = X/L, \quad y = Y/L, \quad s = S/L, \quad b = B/L, \quad c = C/L, \\
&h_f = H_f/L, \quad p = PL^2/EI, \quad q = QL^2/EI, \quad m = ML/EI, \\
&t = (T/L^2)\sqrt{EIg/W}, \quad \Omega = \omega L^2\sqrt{W/EIg}, \\
&k = KL^4/EI, \quad \beta = (e_o a/L)^2, \quad w_{adh} = W_{adh}L^2B_0/EI.
\end{aligned} \tag{2}$$

The nondimensional variables are written in the form

$$\begin{aligned}
&x(s,t) = x_e(s) + x_d(s)\sin\Omega t, \quad y(s,t) = y_e(s) + y_d(s)\sin\Omega t, \\
&\theta(s,t) = \theta_e(s) + \theta_d(s)\sin\Omega t, \quad m(s,t) = m_e(s) + m_d(s)\sin\Omega t, \\
&p(s,t) = p_e + p_d(s)\sin\Omega t, \quad q(s,t) = q_e(s) + q_d(s)\sin\Omega t,
\end{aligned} \tag{3}$$

where subscripts e and d denote "equilibrium" and "dynamic," respectively. The governing equations for equilibrium are

$$\begin{aligned}
&x_e' = \cos\theta_e, \quad y_e' = \sin\theta_e, \quad \theta_e' = m_e, \\
&m_e' = q_e\cos\theta_e - p_e\sin\theta_e, \quad p_e' = 0, \quad q_e' = -w.
\end{aligned} \tag{4}$$

For Cases I–III, $0 < s < 1 - b$.

Small vibrations about equilibrium are examined, and the resulting equations are

$$\begin{aligned}
&x_d' = -\theta_d\sin\theta_e, \quad y_d' = \theta_d\cos\theta_e, \quad \theta_d' = m_d, \\
&m_d' = (q_d - p_e\theta_d)\cos\theta_e - (p_d + q_e\theta_d)\sin\theta_e, \\
&p_d' = \Omega^2 x_d, \quad q_d' = \Omega^2 y_d.
\end{aligned} \tag{5}$$

In addition to the upright loop in Fig. 1(a), the hanging loop shown in Fig. 1(b) is considered. The top of the loop is clamped to a horizontal surface, and the clamping length is denoted C. In Eqs. (4) and (5), w is negative for the hanging loop, and $0 < s < 1 - c$.

3. Experiments

A thin strip of polycarbonate was used to acquire experimental data. The cross-sectional dimensions were: width $B_0 = 25.4$ mm, thickness $H = 0.127$ mm. An independent linear beam bending test suggested a Young's modulus $E = 2.25$ GPa (the polycarbonate manufacturer's specifications gave a range of 2.0–2.4 GPa). The density of the material was measured at 1.18×10^{-3} g/mm^3 (again close to the manufacturer's specifications), thus giving a specific weight of 11.6 kN/m^3, and a specific weight per unit length of $W = 0.0374$ N/m for the strip under consideration. The length of the strip was used as the *control* parameter over an approximate range of $L \approx 150$ mm (a tight loop) to $L \approx 700$ mm (where the loop collapsed onto itself causing self-contact, in the 'up' orientation). The length over which the loop was clamped at the boundary condition was held fixed at $C = 17$ mm. Given the nondimensional expression $w = WL^3/EI$, the slenderness of the strips corresponds to the accessible ranges $\pm w = 14 \rightarrow 1400$.