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Buckling of a stiff thin film on a pre-strained bi-layer substrate

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ABSTRACT

Controlled buckling can impart stretchable mechanics to brittle materials when integrated as thin films on soft, elastomeric substrates. Typical elastomers are permeable to fluids, however, and therefor unable to provide robust barriers to entry of water, for instance, into devices built with the supported thin films. In addition, the mechanical strength of a system dominated by a soft substrate is often unsatisfactory for realistic applications. We show that introduction of a bi-layer substrate yields a robust, high strength system that maintains stretchable characteristics, with a soft layer on top of a relatively stiff layer in the substrate. As a mechanical protection, a soft encapsulation layer can be used on top of the device and the stretchability of the encapsulated system is smaller than that of the system without encapsulation. A simple, analytic model, validated by numerical analysis and FEA, is established for stiff thin films on a bi-layer substrate, and is useful to the design of stretchable systems.

1. Introduction

With the ability to conform to biological tissues and monitor vital physiological signals, stretchable electronics (Song et al., 2014; Lu et al., 2012; Kim et al., 2011a) have the potential to provide a promising platform for biomedical devices as diagnostics and/or therapeutics for clinical purposes (Kim et al., 2011b; Jeong et al., 2013; Webb et al., 2013). Controlled buckling realized by the pre-strain strategy (Jiang et al., 2007; Huang et al., 2005; Allen, 1969; Song et al., 2008; Wang and Zhao, 2014; Zang et al., 2013; Cao et al., 2013; Lu et al., 2007) can generate sophisticated micro- and nano-structures in stretchable electronics (Kim et al., 2011b; Ko et al., 2008; Xiao et al., 2010; Xiao et al., 2009; Duan et al., 2013). Here, as shown in Fig. 1, a stiff film is first transfer printed (Yang et al., 2012; Kim et al., 2012; Cheng et al., 2012) onto a flat, pre-strained elastomeric substrate. When the elastomer returns to its original length upon release of the pre-strain, the film buckles into a wavy geometry, which affords, then, an effective level of stretchable mechanics. With x and z in the film length and thickness directions (Fig. 1a), the out-of-plane displacement w_0 of the buckled thin film can be represented by a sinusoidal function $w_0 = A\cos(kx)$, where A and k are the characteristic amplitude and wave number to be determined. With the film thickness h_f much smaller than the buckle wavelength $2\pi/k$, the thin film is modeled as a beam. Jiang et al. (2007) studied the buckling and post-buckling behaviors of thin film on a single-layer substrate. The total energy of the buckled system consists of bending energy and membrane energy of the film, and elastic energy of the substrate. The bending energy U_b and membrane energy U_m, which will be used in the present study, are given analytically as (Jiang et al., 2007)

$$U_b = \frac{k^4 \bar{E}_f h_f^3 A^2}{48(1+\varepsilon_{pre})^4} L, \quad U_m = \frac{\bar{E}_f h_f}{2}\left[\frac{k^2A^2}{4(1+\varepsilon_{pre})^2} - \frac{\varepsilon_{pre}}{1+\varepsilon_{pre}}\right]^2 L, \qquad (1)$$

where ε_{pre} is the pre-strain applied to the substrate, $\bar{E}_f$ is the plane-strain modulus of the thin film, and L is the film length at the original, unstretched state (Fig. 1b).

The soft substrate plays a key role in the pre-strain strategy. However, being permeable to fluids, the single-layer soft substrate cannot encapsulate the device well, and it is also difficult to integrate with liquid components (Xu et al., 2013). Furthermore, electronics built on unusual substrates, ranging from fabrics to plastic sheets, have great potential for use in biomedical devices (Kim et al., 2009). To enable a stretchable capability in integrated electronics, another soft layer is introduced on top of such unusual types of substrates, resulting in a bi-layer structure, where the soft

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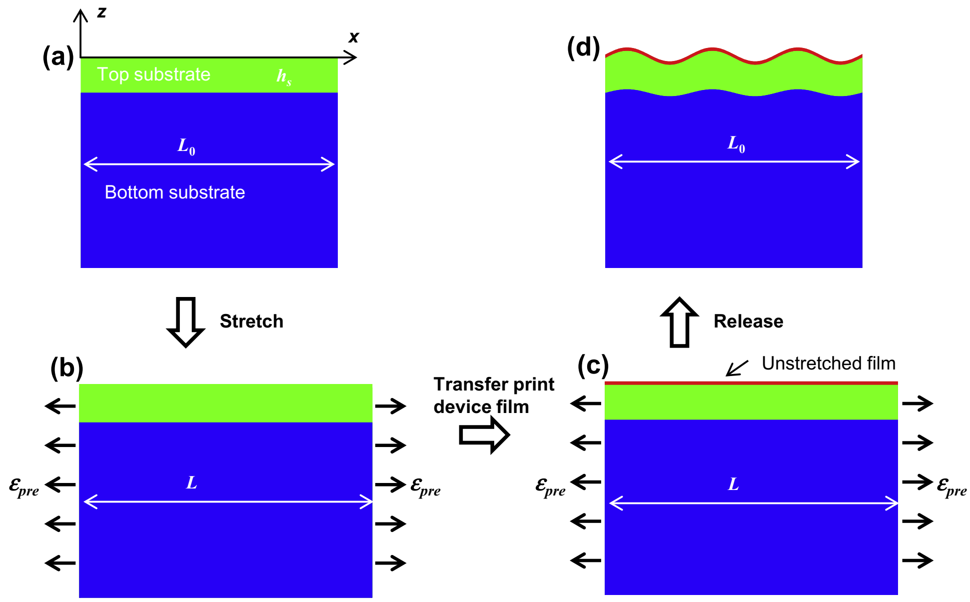

Fig. 1. Schematic illustration of a buckled stiff thin film on a bi-layer substrate.

layer on top facilitates buckling of thin films and the relatively stiff layer at the bottom can significantly enhance the strength of the system (Kim et al., 2009). Careful choice of the bottom layer can further improve the robustness, providing chemical and thermal resistances to the system (García et al., 2010). This bi-layer substrate design is not only critical for use in stretchable electronics, but it can also create opportunities such as integration with liquid components or enhanced strength and robustness in classes of electronics built to dissolve completely after function in the human body or environment via resorption, thereby eliminating the need for recollection (Hwang et al., 2012; Li et al., 2013). To understand the system with a bi-layer substrate, we perform analytic study on buckling and post-buckling behaviors. The elastic energy of the bi-layer substrate is obtained in Section 2. The energy method is then used to study the buckling and postbuckling in Sections 3 and 4, respectively.

2. Elastic energy of bi-layer substrate

Fig. 1 illustrates the pre-strain strategy where a stiff thin film buckles on a bi-layer substrate. The top substrate layer is usually much thicker than the thin film, i.e., its thickness $h_s \gg h_f$ (e.g., h_s = 1 mm and h_f = 100 μm as in experiments). The bottom substrate layer is much thicker than the top layer (e.g., 10 times) Kim et al., 2009, and is therefore modeled as a semi-infinite solid. Similar to thin film buckling on a single-layer substrate, the out-of-plane displacement of buckled thin film on a bi-layer substrate can also be represented by a sinusoidal function $w_0 = A\cos(kx)$. In the system of a stiff thin film buckled on a pre-strained, single-layer substrate, Song et al. (2008) developed a finite deformation theory, which explains the buckled amplitude and wavelength very well. Cheng and Song (2013) further showed that finite geometry change of the thin film dominates in the finite deformation theory, which was also confirmed to apply for the bi-layer substrate in the finite element analysis (FEA). In FEA, a Mooney–Rivlin hyperelastic model was used for both the top and bottom substrate layers. The results were compared with those obtained using linear elastic model, which only shows a slight difference. Therefore, linear elastic model is used in the following analytic study for the bi-layer substrate.

Let u_i and w_i denote the displacements in the x and z directions (Fig. 1a), with $i = 1, 2$ representing the top and bottom layers of the substrate, respectively. The equilibrium equations can be written in terms of the displacements as (Timoshenko and Goodier, 2011)

$$\begin{aligned} &\left(\frac{\partial^2 u_i}{\partial x^2}+\frac{\partial^2 u_i}{\partial z^2}\right)+\frac{1}{1-2\nu_i}\frac{\partial}{\partial x}\left(\frac{\partial u_i}{\partial x}+\frac{\partial w_i}{\partial z}\right)=0,\\ &\left(\frac{\partial^2 w_i}{\partial x^2}+\frac{\partial^2 w_i}{\partial z^2}\right)+\frac{1}{1-2\nu_i}\frac{\partial}{\partial z}\left(\frac{\partial u_i}{\partial x}+\frac{\partial w_i}{\partial z}\right)=0, \end{aligned} \tag{2}$$

where ν_i is the Poisson's ratio of each substrate layer. Let $z = 0$ denote the top surface of the substrate. Continuity of displacement between the thin film and substrate requires

$$w_1|_{z=0} = w_0 = A\cos(kx). \tag{3}$$

The shear at the film/substrate interface is negligible (Huang et al., 2005) because the thin film is much stiffer than both substrate layers (Kim et al., 2009), which gives

$$\left.\left(\frac{\partial u_1}{\partial z}+\frac{\partial w_1}{\partial x}\right)\right|_{z=0}=0. \tag{4}$$

Continuity of displacement and stress requires

$$\begin{aligned} &u_1|_{z=-h_s}=u_2|_{z=-h_s},\quad w_1|_{z=-h_s}=w_2|_{z=-h_s},\\ &\sigma_{z1}|_{z=-h_s}=\sigma_{z2}|_{z=-h_s},\quad \tau_1|_{z=-h_s}=\tau_2|_{z=-h_s}, \end{aligned} \tag{5}$$

where σ_z and τ are the normal and shear stresses.

The displacement field can be expressed as $(u_i, w_i) = [U_i(z)\sin(kx), W_i(z)\cos(kx)]$. Its substitution into Eq. (2) gives the solution

$$\begin{aligned} &U_1 = C_1e^{kz}+C_2e^{-kz}+C_3ze^{kz}+C_4ze^{-kz},\\ &W_1 = -C_1e^{kz}+C_2e^{-kz}+\left(\frac{3-4\nu_i}{k}-z\right)C_3e^{kz}+\left(\frac{3-4\nu_i}{k}+z\right)C_4e^{-kz}, \end{aligned} \tag{6a}$$

$$\begin{aligned} &U_2 = C_5e^{kz}+C_6ze^{kz},\\ &W_2 = -C_5e^{kz}+\left(\frac{3-4\nu_i}{k}-z\right)C_6e^{kz}, \end{aligned} \tag{6b}$$