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Bridging techniques in a multi-scale modeling of pattern formation

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ABSTRACT

Bridging techniques between microscopic and macroscopic models are discussed in the case of wrinkling analysis. The considered macroscopic models are related to envelope equations of Ginzburg–Landau type, but generally, they are not valid up to the boundary. To this end, a multi-scale approach is considered: the reduced model is implemented in the bulk while the full model is applied near the boundary and these two models are coupled with the Arlequin method (Ben Dhia, 1998). This paper focuses on the definition of the coupling model and the transition between two scales. Especially, a new nonlocal bridging technique is presented and compared with another recent one (Hu et al., 2011). The present method can also be seen as a guide for coupling techniques involving other reduced order models.

1. Introduction

Wrinkling phenomenon is one of the major concerns for the analysis, design and optimization of structures (Rossi et al., 2005) and material processing (Abdelkhalek et al., 2010), self-organized surface morphology in biomechanics (Efimenko et al., 2005), pattern formation for micro/nano-fabrication (Bowden et al., 1998), etc. To analyze such phenomena, we propose the use of macroscopic models based on envelope equations as in the field of cellular instability problems (Wesfreid and Zaleski, 1984; Cross and Hohenberg, 1993; Hoyle, 2006). Such macroscopic descriptions are common for Rayleigh–Bénard convection (Newell and Whitehead, 1969; Segel, 1969), buckling of long structures (Damil and Potier-Ferry, 1986; Boucif et al., 1991; Abdelmoula et al., 1992), surface wrinkling of stiff thin films resting on compliant substrates (Bowden et al., 1998; Chen and Hutchinson, 2004; Huang et al., 2004; Huang et al., 2005; Audoly and Boudaoud, 2008; Wang et al., 2008; Brau et al., 2011; Cao and Hutchinson, 2012; Zang et al., 2012), fiber microbuckling and compressive failure of composites (Drapier et al., 2001; Kyriakides et al., 1995; Waas and Schultheisz, 1996), wrinkling of membranes (Rossi et al., 2005; Wong and Pellegrino, 2006; Rodriguez et al., 2011; Lecieux and Bouzidi, 2010; Lecieux and Bouzidi, 2012) and many other instabilities arising in various scientific fields (Wesfreid and Zaleski, 1984; Cross and Hohenberg, 1993). The responses of such systems are often nearly periodic spatial oscillations. Therefore, the evolution can be described by envelope models similar to the famous Ginzburg–Landau equation (Segel, 1969; Damil and Potier-Ferry, 1992; Hunt et al., 2000; Iooss et al., 1989).

A new approach has been recently adopted by Damil and Potier-Ferry, 2006; Damil and Potier-Ferry, 2008; Damil and Potier-Ferry, 2010 to model wrinkling phenomena. The approach is based on the Ginzburg–Landau theory (Wesfreid and Zaleski, 1984; Iooss et al., 1989). In the proposed theory, the envelope equation is derived from an asymptotic double scale analysis and the nearly periodic fields (reduced model) are represented by Fourier series with slowly varying coefficients. This mathematical representation yields macroscopic models in the form of generalized continua. In this case, the macroscopic field is defined by Fourier coefficients of the microscopic field. It has been shown recently that this approach is able to account for the coupling between local and global buckling in a computationally efficient manner (Liu et al., 2012) and it remains valid beyond the bifurcation point (Damil and Potier-Ferry, 2010).

Nevertheless, a clear and secure account of boundary conditions cannot be obtained, which is a drawback intrinsically linked to the use of any model reduction. To solve this problem, a multi-scale modeling approach has been recently proposed in order to bypass the question of boundary conditions (Hu et al., 2011): the full model is implemented near the boundary while the envelope model is considered elsewhere, and these two models are bridged

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by the Arlequin method (Ben Dhia, 1998, 2006, 2008; Ben Dhia and Rateau, 2005). This idea makes it possible to clarify the question of boundary conditions, which keeps the advantages of the two approaches: the envelope model in the bulk makes it possible to simplify the response curves and limit the total number of degrees of freedom; the fine model avoids the cumbersome problem of the boundary conditions being applied to the envelope equation. In this paper, we revisit these coupling techniques between a reference model and a reduced model of Ginzburg–Landau type.

Over the last decade, various numerical techniques have been developed to couple heterogeneous models, e.g. the Arlequin method (Ben Dhia, 1998, 2006, 2008; Ben Dhia and Rateau, 2005) or the bridging domain method (Xiao and Belytschko, 2004). One can couple classical continuum and shell models (Ben Dhia and Rateau, 2005), particle and continuum models (Bauman et al., 2008; Prudhomme et al., 2008; Prudhomme et al., 2012; Bauman et al., 2009; Xiao and Belytschko, 2004), heterogeneous meshes (Ben Dhia and Rateau, 2005; Hu et al., 2009) or more generally heterogeneous discretizations (Ben Dhia and Jamond, 2010; Biscani et al., 2012). For instance, local stresses around the boundary have been computed by coupling 2D elasticity near the boundary and 1D beam model elsewhere (Hu et al., 2009, 2010).

Basically, the Arlequin method aims at connecting two spatial approximations of an unknown field, generally a fine approximation U_f and a coarse approximation U_r. The idea is to require that these two approximations are neighbor in a weak and discrete sense and to introduce Lagrange multipliers in the corresponding differential problems. At the continuous level, a bilinear form must be chosen, which can be L^2-type, H^1-type or energy type (Ben Dhia and Rateau, 2005; Ben Dhia, 2008; Bauman et al., 2008). The first and important application of the Arlequin method is the coupling between two different meshes discretizing the same continuous problem: in this case, the mediator problem should be discretized by a coarse mesh to avoid locking phenomena (Ben Dhia and Rateau, 2005) and spurious stress peaks (Hu et al., 2009). But the two connected problems are not always in the same space, as for instance when dealing with particle and continuous problems. In this case, a prolongation operator has to be introduced to convert the discrete displacement into a continuous one and next a connection between continuous fields is performed (Bauman et al., 2008): this is consistent because the continuous model can be seen as the coarsest one. A similar approach has been applied in the coupling between plate and 3D models. A prolongation operator has been introduced (i.e. from the coarse to the fine level) and the integration is done in the 3D domain but the discretization of the Lagrange multiplier corresponds to a projection on the coarsest problem: thus, in this sense, this coupling of plate/3D is also achieved at the coarse level. In the same spirit, for the coupling between a fine model and an envelope model that is discussed in this paper, the connection should also be done at the coarse level, i.e. between Fourier coefficients. On the contrary, a prolongation operator from the coarse to the fine model had been introduced in the previous paper (Hu et al., 2011) and the connection had been done at this level. Therefore, one can wonder if the imperfect connection observed in Hu et al. (2011) could be improved by introducing a coupling at the relevant level. This paper tries to answer this question by studying again the Swift–Hohenberg equation (Swift and Hohenberg, 1977) that is a simple and illustrative example of quasi-periodic bifurcation. Very probably, the same ideas can be applied to 2D macroscopic membrane models that were recently introduced in Damil et al. (2013). Note that the presented new technique can be considered as nonlocal since it connects Fourier coefficients involving integrals on a period. A similar nonlocal coupling has been introduced in Prudhomme et al. (2012) in the case of an atomic-to-continuum coupling, where the atomic model is reduced by averaging over a representative volume.

The question addressed in this paper is more or less generic in applying bridging techniques to reduced models or multi-scale models. The first papers about the Arlequin method focused on the choice of a bilinear form and its discretization. But in asymptotic multiple scale methods (Sanchez-Palencia, 1980) or in computational homogenization (Feyel, 2003), one clearly distinguishes two independent spatial domains: a macroscopic domain to account for slow variations and a microscopic domain for the rapid variations. Therefore, the connection operators between the two levels have to be clearly defined, as well as the level at which the coupling is achieved. This subject will be discussed in this paper.

The paper is organized as follows. In Section 2, we establish the theoretical framework of the multi-scale approach, using Fourier coefficients as in Damil and Potier-Ferry (2006). Then we derive the macroscopic envelope model together with a critical review of the adopted approach. In Section 3, the reduction methodology to obtain different envelopes has been discussed with the example of an elastic beam subjected to a nonlinear elastic foundation. Section 4 is dedicated to the bridging technique and the discretization. The difference in terms of methodology between the prolongation coupling (Hu et al., 2011) and the reduction-based coupling approach is thoroughly explored. In Section 5, the wrinkling prediction of an elastic beam on a nonlinear elastic foundation is analyzed using the developed nonlocal reduction-based coupling approach. The results are compared to those obtained using the prolongation coupling approach. Conclusions are reported in Section 6.

2. Macroscopic modeling of instability pattern formation

The numerical test considered in this paper is the famous Swift–Hohenberg equation (Swift and Hohenberg, 1977) that corresponds to the problem of a compressed elastic beam coupled with a nonlinear foundation. It has been studied in many papers, for instance in Hunt et al. (1989), Hunt et al. (2000), Damil and Potier-Ferry (2010), Mhada et al. (2012), because it is a very representative example in the study of cellular instabilities. From this microscopic model, a macroscopic envelope model will be presented and studied in the rest of the paper. Among those discussed in Damil and Potier-Ferry (2010), it is not the more accurate, but it is the simplest one and it is able to describe the amplitude modulation of the oscillation. Let us recall that the central point of the paper is a bridging technique used to correct a reduced model near the boundary. This technique has to be robust and it has to play its part for several levels of reduced model.

2.1. Description of the microscopic model

We consider the example of an elastic beam subjected to a nonlinear elastic foundation as shown in Fig. 1. The unknowns are the components $u(x)$ and $v(x)$ of the displacement vector and the normal force $n(x)$, which represents $U(x) = \{u(x), v(x), n(x)\}$. We will study the following set of differential equations:

$$\begin{cases} \frac{dn}{dx} + f = 0, & (a) \\ \frac{n}{ES} = \frac{du}{dx} + \frac{1}{2}\left(\frac{dv}{dx}\right)^2, & (b) \\ \frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) - \frac{d}{dx}\left(n\frac{dv}{dx}\right) + cv + c_3v^3 = 0. & (c) \end{cases} \quad (1)$$

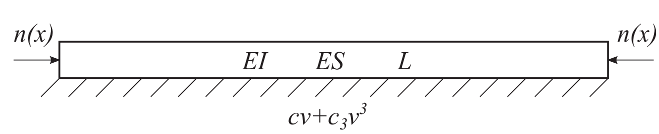

Fig. 1. Sketch of an elastic beam on a nonlinear elastic foundation.