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A planar rod model with flexible cross-section for the folding and the dynamic deployment of tape springs: Improvements and comparisons with experiments

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ABSTRACT

A planar rod model with flexible cross-section has been recently proposed in literature (Guinot et al., 2012). This model is especially suitable for the modeling of tape springs, which develop localized folds due to the flattening of the cross-section. Starting from a complete non-linear elastic shell model, original kinematics assumptions (inspired from the *elastica* model) have been made to describe the important in-plane changes of the cross-section shape. In the present work, the choice of the position of the rod reference line is discussed. This choice plays an important role in the overall behavior because of the large changes of the cross-section shape. We show that the model published in Guinot et al. (2012) can be improved by considering the centerline as the rod reference line. This enhanced model is then validated through quantitative comparisons with experimental results of dynamic deployments taken from literature.

1. Introduction

In its free state, a tape spring can be considered as a straight thin-walled beam with an open circular cross-section of constant transverse curvature. One of the most studied test (Seffen and Pellegrino, 1999) illustrating its behavior is the bending test shown in Fig. 1. Under applied bending rotations at the ends, this structure behaves at first like a beam before the sudden appearance of a localized fold, indicating snap-through buckling. This fold is created by a localized flattening of the cross-section which drastically reduces the moment of inertia and concentrates the bending deformation in the fold area. We shall note that away from the fold, the tape spring remains almost straight and undeformed. Playing with a carpenter's tape measure, one can easily experience the formation of one or several folds, the motion of a fold along the tape, the splitting of a single fold into two or the merging of two folds into one.

Tape springs offer a wide range of compact folded or coiled configurations and thus are an interesting alternative to articulated rigid structures with hinges and bolts for the design of deployment systems. However, since their behavior is sensitive to instabilities and can exhibit a sudden loss of stiffness with largely deformed shapes, the modeling of such structures is a challenging issue.

As mentioned in Guinot et al. (2012), the natural approach for the modeling of tape springs consists in the full computation of a non-linear shell model in the framework of large displacements, large rotations and dynamics (Hoffait et al., 2009; Seffen et al., 2000; Walker and Aglietti, 2007). This approach leads to hard-to-drive and time consuming simulations but provides accurate static and dynamic solutions for any loading configurations and boundary conditions. The difficulties reside mainly in the slenderness of the structure combined with the transverse curvature that lead to a highly flexible structure. The slenderness and the transverse curvature also make the structure sensitive to localized buckling that occurs when overall bending leads to compression effects on the edges of the cross-section.

Considering the particular shape of a tape spring, one can think about an intermediate model based on a thin-walled beam model. The literature is extremely extensive on this topic, from the pioneering work of Vlassov (1962) to the recent developments on the Generalized Beam Theory (Dinis et al., 2009;Silvestre, 2007;

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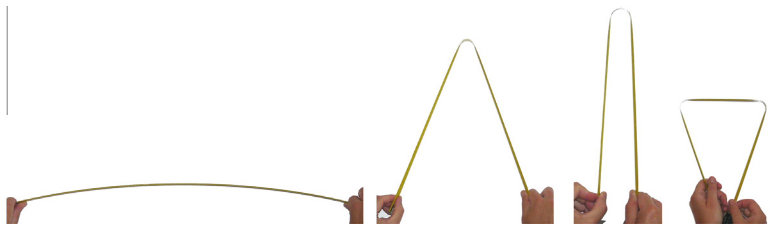

Fig. 1. Folding of a tape spring.

Silvestre et al., 2011;Gonçalves and Camotim, 2009) introduced by Schardt (1994). Compared to all these models, the main originality of the rod model proposed in Guinot et al. (2012) lies in the taking into account of the high flexibility of the cross-section in its plane through a suitable kinematics inspired from the *elastica* theory (Euler, 1744; Goss, 2009), which leads to a reduced number of kinematic parameters. Starting from a non-linear shell model, the main idea underlying the model consists in a parametrization of the cross-section shape (and not of the relative displacements) under the inextensibility assumption of the 'cross-section curve'. This approach has been applied to the folding and dynamic deployment of tape springs in the previous work (Guinot et al., 2012) with a rod model involving only four kinematic parameters. It has been shown that it qualitatively handles the creation of folds, the motion of a fold along the tape and the splitting of a single fold into two. It has however been mentioned that this model has some difficulties to account for snap back phenomena during unloading (see Remark 5 in Guinot et al. (2012)). In the present work, some assumptions on the kinematics are discussed and a new proposal is made to improve the model. It is shown that the choice of the rod reference line is important when large relative displacements in the cross-section are considered. A new proposal is investigated and validated on the classical example treated in Seffen and Pellegrino (1999) and Guinot et al. (2012): the creation of a fold under a pure bending moment prescribed by opposite rotations at ends. The improved model, for which the rod line is taken as the centerline, is able to account for the snap back phenomenon for this example. This improved model is then validated by quantitative comparisons with dynamic deployment experiments presented in Seffen and Pellegrino (1999).

In the following, Section 2 begins to recall the foundations of the model presented in the previous work (Guinot et al., 2012), *i.e.* the basic assumptions about the kinematics that allow to reduce the shell model to a rod one. The choice of the rod reference line is discussed and the case in which the rod line is taken as the centerline is developed. The strain and kinetic energies of the rod model are then obtained. The Hamilton Principle is used to implement the model in the finite element software COMSOL Multiphysics (2011) that performs an automatic differentiation of the energies to obtain the weak formulation of the problem. The next sections are devoted to numerical examples.

In Section 3, a tape spring submitted to opposite cross-section rotations at ends is studied. The overall response (moment versus prescribed rotations at ends) is compared for the previous model, the proposed new model and the shell model. The results show that, contrary to the previous model, the proposed model is able to capture the snap back during the unloading of the prescribed rotations. This result is confirmed by a path-following approach that allows the computation of the whole equilibrium paths, which are consistent with the critical angles at which the snap-through occur for the two rod models. The fold properties are also compared for the shell model and the proposed new model.

In Section 4, the dynamic deployment of a folded tape spring is considered. The improved model is applied to the experiments presented in the work of Seffen and Pellegrino (1999) and quantitative comparisons are analyzed.

2. The rod model

2.1. Kinematic description and basic assumptions

A tape spring is regarded as a shell that can be assimilated to a rod with a thin-walled cross-section. In the initial configuration, the middle surface of the shell is supposed to result from the extrusion of a circular cross-section curve along a straight rod line, as shown in Fig. 2. More precisely, we construct a fixed orthonormal frame $(O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ such that the initial middle surface results from the extrusion along $\mathbf{e}_1$ of an arc of circle contained in the plane $(O, \mathbf{e}_2, \mathbf{e}_3)$. The line defined by $(O, \mathbf{e}_3)$ is chosen to be the axis of symmetry of the arc in the plane $(O, \mathbf{e}_2, \mathbf{e}_3)$ with O an arbitrary point on this axis of symmetry. The initial middle surface of the tape is then symmetric with respect to the plane $(O, \mathbf{e}_1, \mathbf{e}_3)$ by construction of the fixed orthonormal frame $(O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$. The axis $(O, \mathbf{e}_1)$ is chosen to be the rod reference line in the initial configuration.

We naturally introduce a curvilinear coordinate system $(s_1, s_2) \in [0, L] \times [-a, a]$ to map the geometry of the tape, with L the initial length of the tape and $2a$ the initial length of the cross-section curve[1]. The material line defined by $s_1 \in [0, L]$ and $s_2 = 0$ is called the 'bottom line' (see Fig. 2).

At time t, in the deformed configuration, the position of a material point M on the middle surface is given by:

$$\mathbf{OM}(s_1, s_2, t) = \mathbf{OG}(s_1, t) + \mathbf{GM}(s_1, s_2, t), \tag{1}$$

where $\mathbf{OG}$ is the position vector in the deformed configuration of the point which is the intersection of the rod line and the cross-section plane in the undeformed configuration.

The rod model kinematics presented in Guinot et al. (2012) relies on four assumptions:

(i) the cross-section curve remains in a plane after deformation,
(ii) the cross-section plane is orthogonal to the tangent vector of the rod line in the deformed configuration,
(iii) the shape of the tape which is initially symmetric with respect to the plane $(O, \mathbf{e}_1, \mathbf{e}_3)$ remains symmetric with respect to this plane,
(iv) the cross-section curve is considered inextensible and remains circular.

The two first assumptions are the classical hypotheses used in the Euler–Bernoulli beam theory. The symmetry assumption (iii) then involves that the motion of the rod line is restrained to the plane $(O, \mathbf{e}_1, \mathbf{e}_3)$: the displacement of a point G on the rod line is given by the two components $u_1(s_1, t)$ and $u_3(s_1, t)$ and the rotation

[1] The initial length of the cross section curve was set to a in the previous work (Guinot et al., 2012). It is here set to $2a$ to obtain more concise expressions in the following.