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Evaluation of nonlocal approaches for modelling fracture near nonconvex boundaries

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ABSTRACT

Integral-type nonlocal damage models describe the fracture process zones by regular strain profiles insensitive to the size of finite elements, which is achieved by incorporating weighted spatial averages of certain state variables into the stress–strain equations. However, there is no consensus yet how the influence of boundaries should be taken into account by the averaging procedures. In the present study, nonlocal damage models with different averaging procedures are applied to the modelling of fracture in specimens with various boundary types. Firstly, the nonlocal models are calibrated by fitting load–displacement curves and dissipated energy profiles for direct tension to the results of mesoscale analyses performed using a discrete model. These analyses are set up so that the results are independent of boundaries. Then, the models are applied to two-dimensional simulations of three-point bending tests with a sharp notch, a V-type notch, and a smooth boundary without a notch. The performance of the nonlocal approaches in modelling of fracture near nonconvex boundaries is evaluated by comparison of load–displacement curves and dissipated energy profiles along the beam ligament with the results of mesoscale simulations. As an alternative approach, elastoplasticity combined with nonlocal and over-nonlocal damage is also included in the comparative study.

1. Introduction

Among other important factors, the failure process of concrete strongly depends on the meso-structure. Growth and coalescence of microcracks lead to the formation of fracture process zones (FPZ) which transfer stresses by crack bridging and aggregate interlock. Inelastic processes in such zones are commonly modelled by nonlinear fracture mechanics, e.g. cohesive crack models, or by continuum damage mechanics using stress–strain laws with strain softening.

One group of continuum damage mechanics approaches suitable for computational structural analysis are integral-type nonlocal models, which describe the localised fracture process zones by regular strain profiles independently of the size of finite elements (Pijaudier-Cabot and Bažant, 1987; Bažant and Jirásek, 2002). This is achieved by evaluating the stress at each point based on weighted averages of state variables in the vicinity of that point. However, there is no consensus on how the averaging should be adjusted near the physical boundary of the body. Commonly used scaling procedures may result in excessive spurious energy dissipation close to boundaries for notched specimens (Jirásek et al., 2004). In this previous study, it was suggested that the excess in dissipated energy originates from including the contribution of the undamaged material below the notch to the nonlocal variable at a point above the notch, which reduces the damage and, therewith, introduces an artificial strengthening at this point. In alternative approaches which have the potential to reduce this spurious effect, the averaging procedure depends on the distance to boundaries (Bolander and Hikosaka, 1995; Krayani et al., 2009; Bažant et al., 2010), or on the stress state (Bažant, 1994; Jirásek and Bažant, 1994; Giry et al., 2011). Another formulation, which preserves symmetry of the nonlocal weight function, was proposed by Polizzotto (2002), Borino et al. (2002) and Borino et al. (2003) and will be called here the method of local complement. In addition, the spurious energy dissipation might also be affected by the choice of more advanced constitutive models, such as elasto-plasticity combined with nonlocal damage (Grassl and Jirásek, 2006b; Grassl, 2009), where the plastic part could be expected to limit the effective stress and therewith reduce the artificial strengthening described above.

In the present work, a nonlocal damage model with four averaging procedures (representing standard, distance-based, stress-based and local complement averaging) and a plasticity

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model with nonlocal damage based on standard and over-nonlocal averaging procedures (Vermeer et al., 1994; Strömberg and Ristinmaa, 1996; Grassl and Jirásek, 2006) are applied to the modelling of fracture in notched concrete beams subjected to three-point bending. Initially, the models are calibrated by fitting meso-scale analysis results obtained for a problem independent of boundaries (Grassl and Jirásek, 2010). Thus, this calibration is unaffected by the type of averaging procedure used. Then, the nonlocal models are applied to two-dimensional simulations of three-point bending beams with a sharp notch, a V-type notch and a smooth boundary without a notch, for which the averaging procedures close to boundaries are expected to influence the response. The results of the different models are presented in terms of load–displacement curves and dissipated energy profiles, and again compared with meso-scale analyses results.

The meso-scale analyses, which are used to produce reference results to compare the nonlocal model results with, are based on mapping the material properties of individual phases of the heterogeneous meso-structure of concrete on a background mesh (Schlangen and van Mier, 1992). The fracture process of the background mesh is described as the progressive failure of discrete elements, such as lattices of bars and beams (Kawai, 1978; Cundall and Strack, 1979). In lattice approaches, the connectivity between nodes is not changed so that contact determination is simplified. Lattice models are mainly suitable for analyses involving small strains (Herrmann et al., 1989; Schlangen and van Mier, 1992; Bolander and Saito, 1998). In recent years, the discrete element method based on a lattice determined by the Voronoi tessellation has been shown to be suitable for modelling fracture (Bolander and Saito, 1998). The constitutive response for the individual phases can be described by micro-mechanics or phenomenological constitutive models, commonly based on the theory of plasticity, damage mechanics, or a combination of the two. For predominantly tensile loading, an isotropic damage model has shown to provide satisfactory results (Grassl and Jirásek, 2010). Such a model is used here for the meso-scale simulations.

The present meso-scale analyses are based on several assumptions. In the chosen idealisation of the meso-structure only large aggregates are considered, and are embedded in a mortar matrix separated by interfacial transition zones. The aggregates are assumed to be linear elastic and stiffer than the matrix, whereas the interfacial transition zone is assumed to be weaker and more brittle than the matrix. The material constants for the constitutive models of the three phases are chosen by comparing the global results of analyses and experiments assuming certain ratios of the properties of different phases. For instance, aggregates are assumed to be twice as stiff as the matrix, which in turn is twice as strong and ductile as the interfacial transition zone. These chosen ratios are supported by experimental results reported in the literature (Hsu and Slate, 1963) and were used in a recent study on the size effect in notched concrete beams in Grassl et al. (2012) for which the analysis results were in good agreement with experimental data. Furthermore, the present study is limited to two-dimensional plane stress analyses with aggregates idealised as circular inclusions. These are of course strong simplifications. Nevertheless, it is believed that even such an idealised meso-scale model reflects the main features of the mechanical behaviour of concrete as a heterogeneous material with stiff inclusions in a quasi-brittle matrix. In the absence of detailed experimental measurements of the effect of boundaries on the process zone size and energy dissipation density, the meso-scale model is used in the present study as a reference solution against which the nonlocal models are compared.

The meso-scale model reflects the interactions that take place at the material scale and the resulting local redistributions of stress and strain fluctuations. In the nonlocal continuum model, such effects are taken into account in an approximate and simplified way by weighted spatial averaging of an internal variable linked to the inelastic processes.

2. Macroscopic models

In the present section, two macroscopic nonlocal constitutive models based on damage mechanics and on a combination of plasticity and damage mechanics are briefly summarised in Sections 2.1 and 2.2, respectively. Then, the different averaging procedures are described in Section 2.3.

2.1. Damage model

The total stress–strain relationship for the isotropic damage model is

$$\boldsymbol{\sigma} = (1-\omega)\mathbf{D}_{\mathrm{e}} : \boldsymbol{\varepsilon} = (1-\omega)\tilde{\boldsymbol{\sigma}} \tag{1}$$

where $\boldsymbol{\sigma}$ is the total stress tensor, ω is the damage variable, $\mathbf{D}_{\mathrm{e}}$ is the isotropic elastic stiffness tensor based on Young's modulus E and Poisson's ratio ν, $\boldsymbol{\varepsilon}$ is the strain and $\tilde{\boldsymbol{\sigma}}$ is the effective stress tensor. Damage is driven by a history variable κ_{d} and is determined by the damage law

$$\omega(\kappa_{\mathrm{d}}) = \begin{cases} 1 - \exp\left(-\frac{1}{m_{\mathrm{d}}}\left(\frac{\kappa_{\mathrm{d}}}{\varepsilon_{\max}}\right)^{m_{\mathrm{d}}}\right) & , \kappa_{\mathrm{d}} \leqslant \varepsilon_1 \\ 1 - \frac{\varepsilon_3}{\kappa_{\mathrm{d}}}\exp\left(-\frac{\kappa_{\mathrm{d}}-\varepsilon_1}{\varepsilon_{\mathrm{f}}\left[1+\left(\frac{\kappa_{\mathrm{d}}-\varepsilon_1}{\varepsilon_2}\right)^{n}\right]}\right) & , \kappa_{\mathrm{d}} > \varepsilon_1 \end{cases} \tag{2}$$

where

$$m_{\mathrm{d}} = \frac{1}{\ln(E\varepsilon_{\max}/f_{\mathrm{t}})} \tag{3}$$

and f_{t} is the uniaxial tensile strength. Parameter $\varepsilon_{\max}$ is the axial strain at peak stress, and ε_1, ε_2 and n are additional parameters that control the softening part of the stress–strain diagram. Furthermore,

$$\varepsilon_{\mathrm{f}} = \frac{\varepsilon_1}{(\varepsilon_1/\varepsilon_{\max})^{m_{\mathrm{d}}} - 1} \tag{4}$$

and

$$\varepsilon_3 = \varepsilon_1 \exp\left(-\frac{1}{m_{\mathrm{d}}}\left(\frac{\varepsilon_1}{\varepsilon_{\max}}\right)^{m_{\mathrm{d}}}\right) \tag{5}$$

This damage law exhibits pre- and post-peak nonlinearities in uniaxial tension.

The history variable κ_{d}, used in (2) to obtain the damage parameter, represents the maximum level of nonlocal equivalent strain $\bar{\varepsilon}_{\mathrm{eq}}$ reached in the history of the material. It is determined by the loading–unloading conditions

$$f \leqslant 0, \dot{\kappa}_{\mathrm{d}} \geqslant 0, \quad \dot{\kappa}_{\mathrm{d}} f = 0 \tag{6}$$

in which

$$f(\bar{\varepsilon}_{\mathrm{eq}}, \kappa_{\mathrm{d}}) = \bar{\varepsilon}_{\mathrm{eq}} - \kappa_{\mathrm{d}} \tag{7}$$

is the loading function.

The nonlocal equivalent strain is defined as

$$\bar{\varepsilon}_{\mathrm{eq}}(\boldsymbol{x}) = \int_V \alpha(\boldsymbol{x}, \xi)\varepsilon_{\mathrm{eq}}(\xi)\mathrm{d}\xi \tag{8}$$

Here, $\boldsymbol{x}$ is the point at which the nonlocal equivalent strain $\bar{\varepsilon}_{\mathrm{eq}}$ is evaluated as a weighted average of local equivalent strains $\varepsilon_{\mathrm{eq}}$ at all points ξ in the vicinity of $\boldsymbol{x}$ within the integration domain V.

According to the standard scaling approach (Pijaudier-Cabot and Bažant, 1987), the weight function