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Modeling rate-independent hysteresis in large deformations of preconditioned soft tissues

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ABSTRACT

A phenomenological model is proposed for characterizing rate-independent hysteresis exhibited by preconditioned soft tissues. The preconditioned tissue is modeled as an isotropic composite of a hyperelastic component and a dissipative (inelastic) component. Specifically, the constitutive equations are hyperelastic in the sense that the stress is determined by derivatives of a strain energy function. Inelasticity of the dissipative component is controlled by a yield function with different functional forms for the hardening variable during deformation loading and unloading. The constitutive equations proposed in this paper are simple. In particular, they depend on only seven material constants: three controlling the response of the elastic component and the remainder controlling the response of the dissipative component. More importantly, the material constants can be determined to match rather general loading and unloading behavior. It is observed that the hysteretic response of the model compares well with experimental data for passive uniaxial loading/unloading of Manduca muscle. Moreover, the present model treats partial loading and reloading of preconditioned tissue as elastic–plastic response, which is different from the treatment of pseudo-elastic models used in the literature.

1. Introduction

Most biological soft tissues are inhomogeneous, nearly incompressible materials that exhibit nonlinear inelastic (viscoelastic/viscoplastic) response. Many of these soft tissues are reinforced by fiber families which generally consist of collagen and elastin. The material orientation of these fibers along with the fiber constituents play a dominant role in determining the anisotropic mechanical properties of the tissue. Phenomenological models which include specific fiber orientations have been considered in Holzapfel (2001) and Rubin and Bodner (2002). However, for some applications, it is sufficient to model these tissues as isotropic materials.

In general, the material response of the tissue is rate-dependent and inelastic. More specifically, cyclic loading of tissues at constant strain rate between fixed stress or strain limits typically exhibits time-dependent inelastic hysteresis loops that shift with each cycle towards a steady-state hysteresis loop. As an example, Fig. 1. shows the steady-state hysteresis loop for passive cyclic uniaxial stress loading of a Manduca muscle (Dorfmann et al., 2008).

Fung et al. (1972, 1993) observed that this steady-state hysteresis loop is relatively insensitive to the magnitude of the constant strain rate over more than two orders of magnitude of strain rate. This steady-state hysteresis loop characterizes the state of the material which is referred to here as preconditioned. The transitional process towards this preconditioned state is referred to here as preconditioning. Moreover, it is noted that the hysteresis loops of the preconditioned tissue depend on the magnitudes of strain or stress defining the limits of the cycles.

Several researchers have experimentally observed this insensitivity of the response of the preconditioned soft tissues to strain rate. Specifically, the effect was observed for biaxial stretching of rabbit skin in Lanir and Fung (1974a,b) and for excised skin in Pereira et al. (1991). Zheng et al. (1999) found that the effective Young's modulus of limb soft tissue was fairly rate insensitive and Vogel (1972) reported that the strain to failure of rat skin was also rate independent.

Often, the hysteresis loop of the preconditioned tissue is ignored and the tissue is modeled as being a hyperelastic material. Since a hyperelastic material exhibits a single loading/unloading curve it is necessary to decide whether the loading curve, the unloading curve or some average of the two curves in the actual preconditioned hysteresis loop will be used to calibrate the strain energy function for the approximate hyperelastic model. For example, Hendriks et al. (2004, 2006) used a Mooney–Rivlin model for human skin and Shergold and Fleck (2005) and Shergold et al. (2006) used the Ogden model for human skin and pig skin, respectively.

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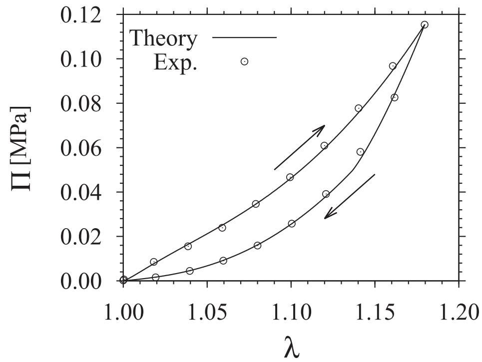

Fig. 1. Comparison of the theoretical model proposed here (Theory) and the experimental data (Exp.) in Dorfmann et al. (2008) for cyclic uniaxial stress loading/unloading of a Manduca muscle.

The most common model used to characterize the hysteresis loop of a preconditioned tissue was proposed by Fung et al. (1972) and Fung (1993), who suggested that the tissue can be modeled by two hyperelastic materials: one characterizing the loading curve and the other characterizing the unloading curve. This material response is called pseudo-elasticity. In particular, a single strain energy function is used with different material constants for the loading and unloading curves.

Within the context of pseudo-elastic models, it is noted that Tong and Fung (1976) developed a strain energy function for modeling the response to biaxial stretching of rabbit skin observed in the experiments in Lanir and Fung (1974a,b). This model had a number of material parameters which were difficult to determine from experimental data and it was found to be too sensitive to changes in the bounds of the biaxial loading. Yin et al. (1986) modified the pseudo-elastic strain energy function in this model to reduce the number of material constants to a nearly "minimum" set needed to match experimental data. Further modification of this pseudo-elastic energy function can be found in Chaudhrya et al. (1998) and Gambarotta et al. (2005).

Dorfmann et al. (2007, 2008) and Paetsch et al. (2012) exploited the isotropic pseudo-elastic model developed by Ogden and Roxburgh (1999) to characterize the passive response of muscle tissue. In this model, the strain energy function is taken in the form $W = W(\boldsymbol{F}, \eta)$, where $\boldsymbol{F}$ is the deformation gradient tensor. The additional variable η is inactive (remains constant) during loading and is a specified function $\eta = \eta(\boldsymbol{F})$ during unloading. The functional form for η is discussed in Ogden and Roxburgh (1999), Dorfmann and Ogden (2003, 2004), Dorfmann et al. (2007) and Paetsch et al. (2012). In particular, the model in Dorfmann and Ogden (2003) is proposed for loading, partial or complete unloading and subsequent reloading and unloading. However, the notion of loading/unloading in this model is unclear and the determination of η for general loading situations is complicated.

A single loading/unloading curve associated with the Mullins effect (Mullins, 1969; Diani et al., 2009) looks identical to that for a preconditioned tissue. In fact, the strain energy function used to model the Mullins effect has the same form $W = W(\boldsymbol{F}, \eta)$. However, for the Mullins effect η is used to characterize damage that only occurs when loading is applied beyond the previous maximum point of loading. Therefore, for the Mullins effect unloading and reloading occur on the same curve with no hysteresis until the material is further damaged. In contrast, unloading and reloading of a preconditioned tissue occur on different curves with hysteresis always being present.

Viscoelastic (Sverdlik and Lanir, 2002) and elastic-viscoplastic (Rubin and Bodner, 2002; Mazza et al., 2005) constitutive equations have been developed which can model the time dependent response of tissues and the process of preconditioning. However, there is still a need for a simple model that characterizes dissipation of the hysteresis loop of a preconditioned tissue. Consequently, the objective of this work is to develop simple isotropic constitutive equations for large deformations of preconditioned biological soft tissues which exhibit rate-independent hysteresis curves and which are valid for general loading histories. In contrast with the standard pseudo-elastic formulation, here the preconditioned tissue is modeled as a composite of a hyperelastic component and a dissipative component. Specifically, the dissipative component is modeled as a rate-independent elastic–plastic material using a yield function, which depends on the elastic distortional deformation of the inelastic component and on a hardening variable. Furthermore, in contrast with the standard uniaxial stress response of metals for cyclic loading, the axial stress in a preconditioned tissue does not change sign during unloading until the axial strain changes sign. To account for this fact, the hardening variable is taken to be a function of the total distortional deformation which vanishes in the unstressed reference state of the tissue. Also, different functional forms for the hardening variable are proposed for deformation loading and unloading, which allow for easy modeling of the hysteresis loop exhibited by preconditioned tissues.

2. Basic equations of the preconditioned tissue

This section briefly reviews constitutive equations for a nonlinear isotropic elastic material and provides background for the developments in the following sections. To this end, it is recalled that a material point $\boldsymbol{X}$ in the fixed reference configuration moves to the point $\boldsymbol{x}$ in the present configuration at time t, with the deformation gradient $\boldsymbol{F}$ and the dilatation J defined by

$$\boldsymbol{F} = \partial \boldsymbol{x} / \partial \boldsymbol{X}, \quad J = \det(\boldsymbol{F}) > 0 \tag{1}$$

Also, the velocity $\boldsymbol{v}$ of a material point, the velocity gradient $\boldsymbol{L}$ and the rate of deformation tensor $\boldsymbol{D}$ are defined by

$$\boldsymbol{v} = \dot{\boldsymbol{x}}, \quad \boldsymbol{L} = \partial \boldsymbol{v} / \partial \boldsymbol{x}, \quad \boldsymbol{D} = \frac{1}{2}\left(\boldsymbol{L} + \boldsymbol{L}^T\right) \tag{2}$$

where the superposed dot denotes material time differentiation holding $\boldsymbol{X}$ fixed.

It can be shown that $\boldsymbol{F}$ and J satisfy the evolution equations

$$\dot{\boldsymbol{F}} = \boldsymbol{L}\boldsymbol{F}, \quad \dot{J} = J(\boldsymbol{D} \cdot \boldsymbol{I}) \tag{3}$$

where $\boldsymbol{I}$ is the second order unity tensor and $\boldsymbol{A} \cdot \boldsymbol{B} = \mathrm{tr}\left(\boldsymbol{A}\boldsymbol{B}^T\right)$ denotes the inner product between two second order tensors $\{\boldsymbol{A}, \boldsymbol{B}\}$.

The preconditioned tissue is considered to be a composite of an elastic component and a dissipative component. In particular, the specific (per unit mass) strain energy Σ of the tissue is modeled as a sum of the specific strain energy Σ_e of the elastic component and the specific strain energy Σ_d of the dissipative component

$$\Sigma = \Sigma_e + \Sigma_d \tag{4}$$

Moreover, the total Cauchy stress $\boldsymbol{T}$ in this model separates additively into two parts

$$\boldsymbol{T} = \boldsymbol{T}_e + \boldsymbol{T}_d \tag{5}$$

where $\boldsymbol{T}_e$ is the stress in the elastic component and $\boldsymbol{T}_d$ is the stress in the dissipative component. Within the context of the purely mechanical theory, the rate of material dissipation is given by

$$\mathcal{D} = \boldsymbol{T} \cdot \boldsymbol{D} - \rho \dot{\Sigma} \geqslant 0 \tag{6}$$

where the conservation of mass relates the mass density ρ_0 in the reference configuration to the mass density ρ in the present configuration