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Closed-form expressions for the macroscopic in-plane elastic and creep coefficients of brick masonry

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ABSTRACT

Approximate expressions for the macroscopic in-plane elastic and creep coefficients of brick masonry with a regular pattern are derived in closed form, using a homogenization approach for periodic media. A microscopic displacement field fulfilling suitable periodicity boundary conditions, and depending on a limited number of degrees of freedom, is formulated over any masonry Representative Volume Element (RVE). According to this field, closed-form expressions for the macroscopic elastic constants are obtained at various degrees of approximation, either using a Method of Cells-type approach, or minimizing the potential energy of the RVE subjected to any given macroscopic stress. Eventually, the results are extended to the description of the global creep behavior of brickwork under service loads, assuming the creep laws of units and mortar to be expressed by Prony series. Using the FE solution as a benchmark, the proposed approach is found to accurately match both the macroscopic constitutive law in linear elasticity and the time evolution of the macroscopic strains of brickwork under sustained macroscopic stress.

1. Introduction

Predicting the global (or macroscopic, or effective) mechanical properties of heterogeneous media from those of the individual components is a goal that many authors have tried to achieve. The advantages of this approach are manifold. When dealing with fiber reinforced composites, for instance, a material with prescribed macroscopic properties can be designed by properly selecting nature, size and orientation of the fibers. In the case of masonry, tests on full scale specimens are costly and require cumbersome devices. Also, in the case of historic buildings, large specimens are usually impossible to take. Thus, performing tests on brick and mortar samples can be a simpler and feasible alternative, provided that reliable formulas to predict the macroscopic properties of masonry are available.

In linear elasticity, several authors have derived expressions for the effective properties of masonry, which is macroscopically orthotropic if made up by regularly spaced units. This was done e.g., by Pande et al. (1989), who exploited results previously obtained by Salamon (1968) for stratified rock to obtain the five macroscopic elastic constants of masonry, assumed to be represented by an equivalent transversely isotropic material. Later, Pietruszczak and Niu (1992) used an approach typical of the mechanics of composite materials, in which head joints are considered as uniformly dispersed weak inclusions and bed joints as continuous planes of weakness. Refined finite element models, such as those proposed by Anthoine (1995) for periodic masonry, or by Cluni and Gusella (2004) for quasi-periodic masonry, are supposed to predict the macroscopic behavior more accurately. As pointed out by Zucchini and Lourenço (2002), the analytical approaches proposed by Pande et al. (1989) or Pietruszczak and Niu (1992) give unreliable predictions if units and mortar have elastic moduli differing by an order of magnitude or more. Accordingly, these authors proposed approximated displacement (and stress) fields for any Representative Volume Element (RVE), defined by a reduced number of variables, and derived the macroscopic elastic stress–strain law by prescribing approximate equilibrium and compatibility conditions at the boundaries of the different parts of the RVE. Their predictions were found to be sufficiently accurate for any ratio of the elastic moduli of brick and mortar by comparisons with FE analyses. This approach allows closed-form expressions for the macroscopic in- and out-of-plane shear moduli to be obtained, whereas Young's moduli and Poisson's ratios are numerically computed.

The macroscopic behavior of masonry beyond the elastic field was mathematically described by Pietruszczak and Niu (1992), taking into account the elastoplastic behavior of the constituents. Their approach allows the macroscopic failure surface to be determined. Alternatively, an approach based on limit analysis

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for periodic brickwork can be used to derive these surfaces without any incremental analysis (Milani et al., 2006). Damage effects in the constituents were taken into account e.g., by Luciano and Sacco (1998), Zucchini and Lourenço (2007), and Shieh-Beygi and Pietruszczak (2008), to describe the brittle post-peak behavior experimentally observed in tests on masonry specimens. More recently, Sacco (2009) derived the macroscopic behavior of 2D brickwork in the nonlinear range by assuming damage and friction effects to develop only in the mortar joints, and applying classical homogenization techniques for periodic media to any RVE.

So far, little attention has been devoted to the mathematical description of the long-term behavior of masonry under sustained loads. Indeed, creep effects in masonry are quite significant, as shown by the laboratory tests carried out by Shrive et al. (1997) or by Sayed-Ahmed et al. (1998), and by the evolution of displacements in full-scale buildings monitored by Hughes and Harvey (1995). According to numerical analyses carried out on 3D finite element models of header bond and Flemish bond masonry walls subjected to sustained vertical load (Taliercio, 2013), the time evolution of the vertical macroscopic creep strain turns out to be nearly unaffected by the brick pattern. A simplified 2D layered model is capable to predict the experimentally measured creep strain (Brooks, 1990) with sufficient accuracy for practical purposes. Under stress conditions other than vertical and uniaxial, a layered model might not give reliable predictions.

Despite the importance of creep phenomena, few authors have tried to formulate mathematical expressions for the macroscopic creep coefficients of brick masonry. Brooks (1986), for instance, extended formulas previously proposed in linear elasticity to mathematically define the creep compliance of masonry under sustained vertical stress, by simply replacing the elastic moduli of the constituents by some effective moduli. Recently, Cecchi and Tralli (2012) proposed an analytical model based on homogenization procedures for periodic media; confining creep phenomena into joints and reducing joints to interfaces, closed-form expressions for the macroscopic creep coefficients can be obtained. The reliability of these expressions has still to be assessed, e.g., by comparisons with accurate finite element analyses.

This paper aims at deriving analytical expressions for the macroscopic elastic and creep coefficients of in-plane loaded masonry with regular brick pattern. The proposed approach has similarities with both the so-called Method of Cells (MoC), originally proposed by Aboudi (1991) to predict the macroscopic mechanical behavior of periodic unidirectional fiber-reinforced composites (in the linear elastic, linear viscoelastic and plastic range), and with the already quoted approach employed by Zucchini and Lourenço (2002).

Whereas in other papers dealing with the same subject closed-form expressions only for some of the macroscopic elastic constants are proposed (see Section 3), here all the in-plane elastic constants (Young's moduli, Poisson's ratio and shear modulus) are given analytical expressions. Another novelty of the proposed approach is that also the macroscopic in-plane creep coefficients can be analytically evaluated.

The layout of the paper is as follows. First, in Section 2.1 the fundamentals of homogenization theory for periodic media are briefly recalled. In particular, in Section 2.2 the Reuss and Voigt bounds for the macroscopic elastic stiffness of any heterogeneous medium are specialized to periodic masonry: these bounds will be used in the following sections to define the possible range of variation of any theoretical estimate for the macroscopic elastic constants. Then, in Section 3 a state-of-the-art is presented on some of the closed-form expressions proposed so far for the macroscopic elastic and creep coefficients of masonry. In Section 4 the original approach proposed to derive the macroscopic properties of masonry is illustrated, and applied to estimate the macroscopic in-plane Young's moduli and Poisson's ratio in Section 4.1 and the in-plane shear modulus in Section 4.2. It is shown how the expressions obtained in the elastic field can be extended to describe the macroscopic creep behavior of masonry. In Section 5 the accuracy of the analytical expressions derived in Section 4 is assessed through comparisons with the results of refined finite element analyses and with closed-form expressions available in the literature, recalled in Section 3. Finally, in Section 6 the main findings of the work are summarized and possible future developments are outlined.

A detailed list of the symbols used in the text is provided in Appendix B.

2. Homogenization theory for periodic media: a brief outline

In this section, some fundamental concepts of homogenization theory for periodic media are briefly recalled. Readers are referred e.g., to Nemat-Nasser and Hori (1993) for a detailed discussion on this subject.

2.1. Macroscopic elastic tensor

When dealing with heterogeneous media, it is customary to replace the real medium by a 'homogenized' one and define its global (or macroscopic) properties through the analysis of a Representative Volume Element (RVE). The RVE is the smallest part of the real medium that contains all the information required to completely characterize its average mechanical behavior. If the medium is periodic (as in the case of brickwork with a regular pattern), a single 'unit cell' (V) can be used as RVE. Fig. 1(a) shows a possible choice for the RVE of running bond or header bond brickwork. The macroscopic constitutive law establishes a relationship between macroscopic stresses ($\underline{\underline{\Sigma}}$) and strains ($\underline{\underline{E}}$), which are defined as the volume averages, over the RVE, of the corresponding microscopic variables:

$$\underline{\underline{\Sigma}} = \frac{1}{|V|}\int_V \underline{\underline{\sigma}}(\underline{x})dV, \quad \underline{\underline{E}} = \frac{1}{|V|}\int_V \underline{\underline{\epsilon}}(\underline{x})dV. \tag{1}$$

In particular, as the microscopic strain $\underline{\underline{\epsilon}}$ must be periodic, neglecting rigid body motions the microscopic displacement field $\underline{u}$ must be of the form

$$\underline{u} = \underline{\underline{E}} \cdot \underline{x} + \underline{\tilde{u}}, \tag{2}$$

where $\underline{\tilde{u}}$ is periodic over V. Fig. 1(b) and (c) shows RVE's deformed according to Eq. (2) under macroscopic vertical compression and shear, respectively.

In linear elasticity, the macroscopic constitutive law can be alternatively written as $\underline{\underline{\Sigma}} = \underline{\underline{\underline{\underline{D}}}}^{hom} : \underline{\underline{E}}$, or as $\underline{\underline{E}} = \underline{\underline{\underline{\underline{C}}}}^{hom} : \underline{\underline{\Sigma}}$. $\underline{\underline{\underline{\underline{D}}}}^{hom}$ denotes the macroscopic stiffness tensor, and $\underline{\underline{\underline{\underline{C}}}}^{hom}$ its inverse (also called macroscopic flexibility tensor). In the 2D case, assuming masonry to be macroscopically orthotropic, both tensors are defined by four independent elastic constants. From here onwards, x_1 denotes an axis parallel to the bed joints and to the mid-plane of the wall, x_2 an axis parallel to the head joints and to the mid-plane of the wall, and x_3 an axis parallel to the wall thickness (see Fig. 1(a)). Under in-plane stress, and assuming plane stress conditions, the macroscopic elastic constitutive law can be written as

$$\begin{pmatrix} E_{11} \\ E_{22} \\ 2E_{12} \end{pmatrix} = \begin{pmatrix} 1/E_1 & -\nu_{21}/E_2 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{pmatrix} \begin{pmatrix} \Sigma_{11} \\ \Sigma_{22} \\ \Sigma_{12} \end{pmatrix}, \tag{3}$$

where E_1 and E_2 are macroscopic Young's moduli, ν_{12} and ν_{21} are macroscopic Poisson's ratios (with $\nu_{12}/E_1 = \nu_{21}/E_2$), and G_{12} is the macroscopic in-plane shear modulus.