International Journal of Solids and Structures 51 (2014) 2964–2978

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A nonlinear finite element model for the stress analysis of soft solids with a growing mass

Yin Liu, Hongwu Zhang *, Yonggang Zheng, Sheng Zhang, Biaosong Chen

State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Faculty of Vehicle Engineering and Mechanics, Dalian University of Technology, Dalian 116024, PR China

ARTICLE INFO

Article history:
Received 25 December 2013
Received in revised form 20 March 2014
Available online 24 April 2014

Keywords:
Mass growth
Finite element method
Soft material
Multiplicative decomposition
Free growth

ABSTRACT

The paper presents a new finite element (FE) model for the stress analysis of soft solids with a growing mass based on the work of Lubarda and Hoger (2002). Contrary to the traditional numerical methods emphasizing on the influence of growth on constitutive equations, an equivalent body force is firstly detected, which is resulted from the linearization of the nonlinear equation and acts as the driver for material growth in the numerical aspect. In the algorithm, only minor correction on the traditional tangent modulus is needed to take the growth effects into consideration and its objectivity could be guaranteed comparing with the traditional method. To solve the resulted equation in time domain, both explicit and implicit integration algorithms are developed, where the growth tensor is updated as an internal variable of Gauss point. The explicit updating scheme shows higher efficiency, while the implicit one seems to be more robust and accurate. The algorithm validation and its good performance are demonstrated by several two-dimensional examples, including free growth, constrained growth and stress dependent growth.

1. Introduction

Soft materials with a growing mass have drawn wide attentions in recent years, which are complex mechanical materials with typical nonlinear, anisotropic, large strain and inhomogeneous behaviors (Humphrey, 2003; Menzel and Kuhl, 2012).

In the past two decades, continuum theories handling growth phenomena of soft solids within the framework of thermodynamics are well established (Menzel and Kuhl, 2012). The key kinematic assumption is the multiplicative decomposition of deformation gradient tensor into a growth part and an elastic deformation part, which was first adopted in growth mechanics by Rodriguez et al. (1994) to analyze the growth-induced residual stress of biomaterials. The decomposition was originally introduced by Kröner (1959), and then Lee (1969) and Stojanović et al. (1964) made applications to elasto-plastic and thermoelasticity problems at finite strain, respectively. Owing to their work, growth models based on the multiplicative decomposition are predominant in the current literatures for material growth, such as the contributions by Maugin and Imatani (2003), Epstein and Maugin (2000), Kuhl and Steinmann (2003), Lubarda and Hoger (2002), Loret and Simões (2005), Ganghoffer (2013) and Ganghoffer et al. (2014). Other recent developments could be referred to Ciarletta and Maugin (2011) and Ciarletta et al. (2011). In their work, a second gradient theory for material growth and remodeling is developed, which shows that evolution of structural changes is governed by Eshelby-like stress and hyperstress. In addition, another noticeable progress was made by Yavari (2010), in which the growth mechanics was formulated within the context of differential geometry. Ganghoffer and Sokolowski (2014) proposed a micromechanical approach in which the volumetric and surface growth is described in the framework of shape optimization.

Based on the multiplicative decomposition, many theoretical explorations towards engineering applications have been carried out in recent years, such as growth of soft material under geometrical constraint (Ben Amar and Ciarletta, 2010), growing arteries (Goriely and Vandiver, 2010) and growth-induced instabilities and folding in tubular organs (Ben Amar and Goriely, 2005; Ciarletta and Ben Amar, 2012). These works shed light on some basic physical mechanisms of phenomena in growing materials and may provide new methodologies for the studies of growth mechanics. For further background information of growth mechanics, readers are referred to state-of-the-art reviews by Taber (1995), Humphrey (2003), Ambrosi et al. (2011), Cowin (2011) and Jones and Chapman (2012) and the references therein. However, as indicated in the above referred papers, analytical methods

* Corresponding author. Tel./fax: +86 411 84706249.
E-mail address: zhanghw@dlut.edu.cn (H. Zhang).

http://dx.doi.org/10.1016/j.ijsolstr.2014.04.010
0020-7683/

may be helpless to describe the evolution of growth which is very complicated but important to understand it better.

In the numerical aspect, there are also many elaborations based on the multiplicative decomposition. An analogy between the concept of thermal strain in finite deformation and the growth tensor has been made by Feng and coworkers. Based on the idea, they have studied the surface folding of esophageal mucosa (Li et al., 2011a) and surface wrinkling of core–shell structure (Cao et al., 2011; Li et al., 2011b) with the aid of commercial software, Abaqus. Their works make the growth modeling go a big step from labs to engineering applications. However, mass generation or sorption and their influence on material constitutive equations are not fully considered in their model. It seems to be unknown that whether the critical growth factor is in accordance with the theoretical one or not, in spite that good results of buckling mode could be observed in their work. Based on Lubarda and Hoger's work (2002), Himpel et al. (2005) proposed a computational framework to model isotropic multiplicative growth within an implicit nonlinear finite element setting, in which the isotropic stretch ratio is introduced as an internal variable at the integration points. A new incremental tangent modulus is developed in the intermediate or current configuration to reflect the growth influence on the material. Based on the proposed framework, they made several noticeable attempts towards patient specific simulations, such as stress-induced arterial wall growth (Kuhl et al., 2007), cardiac growth, dilation and wall thickening (Göktepe et al., 2010a,b), growing skins beyond the physiological limit (Buganza Tepole et al., 2012) and so on. A summary of their model and related applications could be referred to (Menzel and Kuhl, 2012; Kuhl, 2013). However, the tangent modulus seems to be unsymmetric in some cases (Himpel et al., 2005), which may lead to lower computational efficiency for large scale problems. Meanwhile, the objectivity of tangent modulus is not expounded in detail. In addition, Menzel (2007) developed a remodeling framework for orthotropic continua, in which the reorientation of fiber families in multiplicative anisotropic growth was illuminated. More recently, a stress-induced volumetric material growth model in thermoelastic continua was developed by Vignes and Papadopoulos (2010), where the material growth is regulated by a three-surface activation criterion and corresponding flow rules. There are many other works conducted within the context of mixture theory. For example, Garikepati et al. (2004) proposed a coupling model of mass transport and mechanics, in which the mass change amongst the individual species rather than for a mass exchange with environment was considered. For an improvement version considering the interactions between transport and mechanics is referred to Narayanan et al. (2009). Similarly, Davol et al. (2008) also made many attempts towards a general thermomechanical theory for a mixture of growing elastic constituents with aim to model cartilage growth. Within the framework of the theory of porous media, Ehlers et al. (2009) provided a continuum-biomechanical approach for biological tissue, which extends the classical theory of mixture towards immiscible materials. Though so many attempts have been made for the numerical modeling of growth phenomena, it is still hard to answer which one is the better. Since comparisons of the results with the experiments or analytical solutions are rare, even for the simple case. Actually, many results could only be explained qualitatively in the current stage.

This contribution aims to develop a new computational framework for modeling growth phenomena of soft material following Lubarda and Hoger's work (2002). Unlike the algorithm developed by Himpel et al. (2005) and their follow-up works, which incorporates growth effects into tangent modulus, we introduce the objective Oldroyd stress rate to linearize the nonlinear equation in the current configuration, which is a common practice in nonlinear finite element method (Crisfield, 1997; Wriggers, 2008). Following this line of thought, a new equivalent body force is emerged in the linearized rate equation, which is related to the growth tensor, growth rate, stress, mass generation rate, etc. The force is assured to act as the driver to make the material grow in the numerical aspect. To the authors' knowledge, such kind of numerical implementations for the soft matter with a growing mass have not yet been implemented. The tangent modulus in our model is symmetric, which could be deduced by a subtle change of modulus in the classical FE model without growing mass. The growth tensor and its rate are updated at the Gauss points as internal variables and are used to calculate the growing body force. The final equation is time-dependent and the corresponding integration algorithms are developed. In case the growth rate is dependent on the mechanical quantities or the growth factor itself, an implicit integration scheme is implemented to stabilize the solution.

The paper is organized as follows. In Section 2, the theoretical background for growth mechanics is briefly reviewed and an objective constitutive law is proposed for the linearization of the equation. In Section 3, the finite element implementation and its linearized version are presented in detail. The mechanical variables are updated via a prediction–correction algorithm and the equivalent driving force should be calculated in the prediction step. Both explicit and implicit schemes are explained to integrate the growth tensor. Section 4 presents several numerical examples, including free growth, constrained growth and stress dependent growth, so as to validate the algorithm. Some analytical analyses on the examples and comparison between the implicit and explicit algorithm are also conducted in the section. Finally, conclusions are made in Section 5.

2. Theory

2.1. Kinematics

Consider a continuous elastic body $\mathcal{B}$ described by a set of material points $\mathbf{X}$ in the reference configuration. The motion of body $\mathcal{B}$ is given as a one-parameter family of configuration $\varphi_t : \mathcal{B} \to \mathbb{E}^3$ and the location of point $\mathbf{X}$ at time t becomes

$$\mathbf{x} = \varphi_t(\mathbf{X}) \tag{1}$$

The description of the body at point $\mathbf{x}$ is referred to as the current configuration. Let $\mathbf{F} = \partial\mathbf{x}/\partial\mathbf{X} = \mathrm{Grad}\,\mathbf{x}$ be the deformation gradient tensor, then its multiplicative decomposition, as shown in Fig. 1, is introduced as

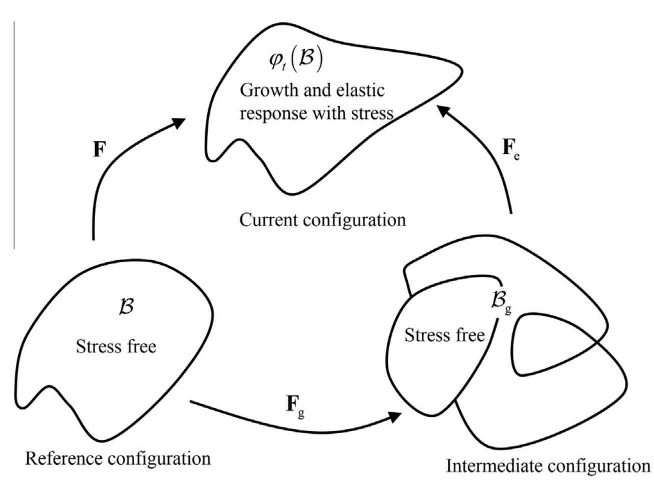

Fig. 1. Decomposition of deformation gradient tensor into a growth part and an elastic part.