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Dynamic response of a growing inclusion in a discrete system

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ABSTRACT

The propagation of a semi-infinite line defect, contained in an infinite square-cell lattice is considered. The defect is composed of particles lighter than those in the ambient lattice and it is assumed this defect propagates with constant speed. Dispersion properties of the lattice are related to waves generated by the propagating defect. In order to determine these properties, the Wiener–Hopf technique is applied. Additional features, related to localisation along the defect are also identified. Analysis of the dispersion relations for this lattice, from the kernel function inside the Wiener–Hopf equation, is carried out. The solution of the Wiener–Hopf equation is presented for the case when an external load is applied corresponding to an energy flux at infinity.

1. Introduction

Large deformations occurring in composite materials can lead to regions of plastic flow, where the formation of line inclusions can occur (Özturk et al., 1991). When waves propagate through a composite, defects within the composite can lead to localised defect modes i.e. large deformations around the defects (Movchan et al., 2007). Such large deformations can also drive the growth of a defect or an inclusion through a composite. In this article, the model for a growing line inclusion within a discrete lattice, having particles which are lighter than those in the ambient lattice, is considered. The model provides information of dispersion properties of the lattice, and at particular frequencies, waves travelling through the lattice may propagate only along the inclusion or the ambient lattice. The former situation may enhance the growth of the inclusion through the lattice and therefore it is important to determine the frequencies which generate such vibrating modes.

Lattice models describing defects propagating through discrete structures have been the subject of many articles. In Slepyan (2001a), the scalar problem of a dynamic crack moving with constant speed through a homogeneous square-cell lattice is studied. The dynamic crack can be considered as a sequential removal of neighbouring bonds, along a row in the lattice, caused by feeding waves which supply energy to the crack front bond (Slepyan, 2002). When this bond breaks, energy is released in the form of dissipative waves which carry energy away from the front. The problem of a fault moving through an elastic triangular lattice was studied in Slepyan (2001c). For both the square and triangular lattices, the wave dispersion properties for the lattice can be deduced in explicit form.

Apart from the introduction of the dynamic crack, additional inhomogeneities which also affect the dispersion properties in a lattice or a continuum can be considered. An overview of several models that reveal the dispersive nature of waves in continua and periodic structures was presented in Movchan et al. (2012), which also includes comparisons of the filtering effects of a bi-atomic chain and a high-contrast periodic continuum. The problem of a structured interface contained in a continuum was also solved and the reflection and transmission due to the interface were analysed. In Mishuris et al. (2009a), a square-cell lattice containing a propagating crack, and composed of rows of particles having contrasting mass, was analysed. The influence of this additional inhomogeneity on the energy dissipation due to crack propagation was also considered.

An inhomogeneous triangular lattice, composed of bonds with contrasting stiffness in the principal lattice directions, can be found in Nieves et al. (2013). This is based on a similar model of that developed in Slepyan (2001c).

In all the above linear models, interaction between the particles of the lattice is assumed to occur between the nearest neighbours and the bonds connecting particles are assumed to be massless. From these models, other useful properties of the fault, such as the energy release rate may also be obtained.

In some of these models (Colquitt et al., 2012; Mishuris et al., 2009b), a fracture criterion has been used to obtain information about real time progression of the crack. In particular, this can

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reveal information about the formation and coalescence of voids ahead of the crack front bond. An inhomogeneous square-cell lattice containing a propagating fault, subjected to a remote sinusoidal load was considered in Mishuris et al. (2009b). When a fracture criterion is imposed on the crack path bonds, the problem becomes non-linear. Numerical simulations have been used to calculate average crack speeds and a good comparison has been found with estimates from dispersion diagrams. Similar calculations for an inhomogeneous triangular lattice have been carried out in Colquitt et al. (2012). Extensions of this analysis have been used to predict the propagation of an edge crack in a structured thermo-elastic solid, where the crack propagation is driven by rapid change in boundary temperature near the interface the crack emanates from and the elastic waves generated by shocks induced by the rapid temperature change (Carta et al., 2013).

It is also possible to find continuum models where the influence of the material's microstructure may be studied. The theory of couple-stress elasticity introduced in Koiter (1964), is designed to take into account characteristic lengths associated with bending and torsion within a material, and this allows the microstructural properties of a material to be studied. In Mishuris et al. (2012), this model was used to study a Mode III semi-infinite crack propagating at constant speed through a continuum that has a microstructure. The Wiener–Hopf technique was also applied to solve this problem and illustrations showing the effect of the material's bending and torsion characteristic lengths on the crack's propagation were presented. The method was also extended to the case of a Mode III crack propagating through an elastic material having a high rotational inertia introduced through its microstructure in Morini et al. (2013). There, the influence of the micro-rotational inertia on the energy dissipation due to crack propagation was also analysed, as this dissipation can affect displacements in the vicinity of the crack tip. It was shown that this rotational inertia may enhance or diminish the energy dissipation associated with the propagating crack, in comparison to its energy dissipation in the model of classical elasticity.

The influence of the microstructure can also be determined from models of discrete structures which have the same effective properties as the corresponding homogenised medium. As an example, the effect of the microstructure on the crack-tip behaviour for an edge crack contained in a slab, which is subjected to a sinusoidal temperature load, was also investigated in Colquitt et al. (2012). For this quasi-static thermoelastic problem, it was shown that the "effective stress intensity factor" for the edge crack in the triangular lattice was lower than that in the homogenised lattice. This indicates that the introduction of microstructure in a solid may lessen the possibility of crack propagation.

For the high-frequency regime of the applied load, a theory of asymptotic homogenisation has been developed in Craster et al. (2010). A continuum model is constructed from the standing wave modes for the lattice problem, and the solution of this continuum problem provides information about the microstructure.

The propagation of an inclusion within a lattice has also been considered in Slepyan (2001b). Here, instead of the removal of subsequent bonds in a row, giving rise to a crack, these bonds undergo a transition in phase, i.e. a jump in stiffness, when the elongation of the bonds reaches a critical point. Another way to interpret the propagation of an inclusion within the lattice is to assume that the inclusion is composed of particles of different mass to those in the ambient lattice. This is discussed in the present paper. The required dispersion properties are obtained from Colquitt et al. (2013), where eigenfrequencies and eigenmodes, corresponding to localised defect modes, for a finite line defect contained in a square lattice were computed. For a long line defect, these eigenfrequencies have been shown to lie in the range of frequencies predicted by the model for the infinite line defect (Osharovich and Ayzenberg-Stepanenko, 2012). An illustrative example given in Colquitt et al. (2013), shows a computation for the eigenfrequencies for a line defect containing 20 particles having contrasting mass compared to the ambient lattice. On Fig. 1, these frequencies are shown by dashed horizontal lines. The solid curve in this figure, depicts the dispersion curve of the infinite line defect model of Osharovich and Ayzenberg-Stepanenko (2012), and it is possible to see from this figure that this model can be used to predict the complete range of the frequencies for a long finite line defect. The same dispersion relation is also encountered in the work presented here. It is also noted that the density of the eigenfrequencies for the finite line defect shown in Fig. 1, increases as we approach the frequencies of the standing wave modes for the infinite line defect, where the homogenised model for this defect is applicable.

The diffraction of waves due to a semi-infinite line of rigid small inclusions within a continuum, called a grating, has been studied in Hills and Karp (1965). This problem has been solved using the Wiener–Hopf technique (Noble, 1958; Hochstadt, 1989) and this solution has used to describe resonance modes for the grating as well as the dispersive nature of this line defect (Hills, 1965; Hills and Karp, 1965).

For finite length rigid line inclusions contained in an elastic material, the complete solution has been obtained for this problem in the case of when the ambient matrix is a bimaterial and the rigid line inclusion is located along the interface of the two materials Ballarini, 1990. Here the strength of the singularities found in the solution near the tips of the inclusion have been determined and compared with those in the problem of when there is a crack along the interface in the bimaterial. A similar analysis has been given in Dal Corso et al. (2008) and Bigoni et al. (2008), where the model for a prestressed elastic material containing a rigid line inclusion under Mode I and II loading has been considered. The influence of the inclusion on the fracture patterns in this problem was also studied and the analytical results were shown to give a good agreement with those obtained in experiments.

A semi-infinite line defect is considered here which propagates through a square-cell lattice. The structure of this article is as follows. In Section 2, the problem of the propagating semi-infinite fault, composed of particles with reduced mass compared to the ambient infinite square lattice, is analysed. The description of this problem and the main notations are given in Section 2.1. In Section 2.2, the dynamic equations are introduced for the

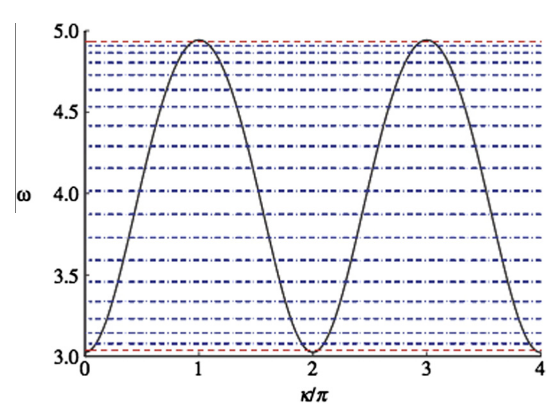

Fig. 1. Example computation from Colquitt et al. (2013), showing the comparison of the eigenfrequencies (ω) computed for a line defect composed of 20 particles with different mass compared to that of the particles in the ambient lattice (indicated by horizontal dashed lines) and the curve given for the infinite line defect in the lattice (Osharovich and Ayzenberg-Stepanenko, 2012). Both are shown as functions of the normalised wavenumber κ/π. Here the ratio of the mass of the particles in the line defect to the mass of the particles in the ambient lattice is 0.25.