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Stiffness reduction of cracked general symmetric laminates using a variational approach

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ABSTRACT

In this paper, stiffness reduction of general symmetric laminates containing a uniform distribution of matrix cracks in a single orientation is analyzed. An admissible stress field is considered, which satisfies equilibrium and all the boundary and continuity conditions. This stress field has been used in conjunction with the principle of minimum complementary energy to get the effective stiffness matrix of a cracked general symmetric laminate. Natural boundary conditions have been derived from the variational principle to overcome the limitations of the existing variational methods on the analysis of general symmetric laminates. Therefore, the capability of analyzing cracked symmetric laminates using the variational approach has been enhanced significantly. It has been shown that the method provides a rigorous lower bound for the stiffness matrix of a cracked laminate, which is very important for practical applications. Results derived from the developed method for the properties of the cracked laminates showed an excellent agreement with experimental data and with those obtained from McCartney's stress transfer model. The differences of the developed model with McCartney's model are discussed in detail. It can be emphasized that the current approach is simpler than McCartney's model, which needs an averaging procedure to obtain the governing equations. Moreover, it has been shown that the existing variational models are special cases of the current formulation.

1. Introduction

Matrix cracking has long been recognized as the first damage mode observed in composite laminates under static and fatigue tensile loading. Matrix cracks, also known as intralaminar cracks, do not usually cause the final failure of a laminate, but may significantly impair the effective properties of the composite and serve as a source for other damage modes initiation, such as delamination and microcracking in the adjacent plies.

Matrix cracks and their effects on material properties degradation have gained much attention both experimentally, numerically and analytically due to their practical importance, see reviews (Nairn, 2000a; Berthelot, 2003; Kashtalyan and Soutis, 2005; Kaddour et al., 2013). Although most research works have been done on cross-ply laminates, the problems of stiffness degradation and damage accumulation in more general lay-ups have also been addressed. Generally, approaches to the problem include different modifications of shear lag model (Nairn and Mendels, 2001; Yokozeki and Aoki, 2005) (to mention a few of them), stress-based variational model (Hashin, 1985), stress transfer model (McCartney, 1992), displacement-based variational model (Zhang and Minnetyan, 2006), continuum damage mechanics approach (Talreja, 1985; Barbero et al., 2013a; Jalalvand et al., 2013), discrete damage mechanic (Barbero et al., 2011), synergistic damage mechanic (Singh and Talreja, 2008), numerical methods such as finite element (Yuan and Selek, 1993) and finite strip method (Li et al., 1994), etc.

Among the approximate analytical and numerical models, the stress-based variational approach (Hashin, 1985, 1986, 1987), stress transfer model of McCartney (McCartney, 1992, 2000; McCartney and Pierse, 1997; Katerelos et al., 2006) and discrete damage mechanics of Barbero (Barbero et al., 2011, 2014; Barbero and Cosso, 2014) have shown to accurately predict stiffness reduction and crack evolution of symmetric laminates (Nairn, 2000b; Vinogradov and Hashin, 2005; Barbero et al., 2013b). These approaches are interesting because the material properties of the damaged laminate depend exclusively on the crack density and no additional parameters or functions are required (Barbero and Cortes, 2010). Nevertheless, the stress analysis of cracked laminates needed in stiffness reduction and damage evolution is generally a complex task (Singh and Talreja, 2010).

Although, McCartney (2000, 2005) and Barbero (Barbero and Cortes, 2010; Barbero et al., 2013b) have extended their models to analyze stiffness reduction of general symmetric laminates with

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arbitrary stacking sequence and even containing cracks in more than one orientation, the variational approach has mostly been used for treating either cross-ply laminates (Hashin, 1985, 1986, 1987; Nairn, 1989; Nairn and Hu, 1992; Varna and Berglund, 1992; Rebiere et al., 2001; Kuriakose and Talreja, 2004) or other symmetric laminates have been reduced to cross-ply by averaging out the off-axis plies (Joffe and Varna, 1999; Li and Lim, 2005). Recently, Vinogradov and Hashin (2010) have extended the capability of the variational approach to analyze stiffness reduction of $[\theta_m{}^{(1)}/\theta_n{}^{(2)}]_s$ laminates containing matrix cracks in the middle ply. In addition, it should be noted that the mathematical model for all mentioned variational works involves effectively only two layers, one cracked and one un-cracked, representing a three-layered laminate after applying symmetry considerations. Therefore, these models do not have the capability of analyzing stiffness reduction for the symmetric laminates with arbitrary stacking sequence and multiple cracked and un-cracked layers, due to the lack of boundary conditions for un-cracked layers. Li and Hafeez (2009) have overcome this drawback for cross ply laminates by introducing some boundary conditions as an outcome of variational procedure and translational symmetry (Li et al., 2009), called natural boundary conditions, in the terminology of variational calculus. It is noted that their model (Li and Hafeez, 2009) has only considered cross ply laminates under axial loading due to the assumed admissible stress field. More recently, Hajikazemi and Sadr (2014) have developed a variational model to analyze the stress field of cracked symmetric laminates under general in-plane loading. As a result, the applicability of the variational approach has been extended fundamentally for considering multiple layers symmetric laminates with arbitrary stacking sequence. However, they only compared their developed stress field with the available stress results obtained from other variational approaches, for the case of cross-ply laminates.

In the current research work, as distinct from the latter publication by the same authors (Hajikazemi and Sadr, 2014), stiffness reduction of cracked general symmetric laminates with arbitrary stacking sequence and multiple cracked and un-cracked layers is analyzed. Therefore, the recently developed stress field (Hajikazemi and Sadr, 2014) has been used in conjunction with the principle of minimum complementary energy to get the effective stiffness matrix of a cracked general symmetric laminate. In this regard, it has been revealed that the present method provides a rigorous lower bound for the stiffness matrix of a general symmetric cracked laminate, which is very important for practical applications. Moreover, a systematic way of evaluating governing equations is developed, which is completely analytical. Therefore, the model could enjoy the advantages of ply refinement technique where each layer of the laminate is subdivided into plies having the same properties in order that important through the thickness variations of the stress components could be taken into account. Stiffness reduction of symmetric laminates obtained by the suggested approach is in excellent agreement with experimental data available in the literature and with those obtained from McCartney's stress transfer model. The differences of the developed model with McCartney's model are discussed in detail. The study of the results has revealed that the McCartney's stress transfer solution can also be derived using a variational method that is not a Reissner method. Moreover, it can be emphasized that the current approach is simpler than McCartney's model, which needs an averaging procedure to obtain the governing equations. It has been shown that the existing variational models are special cases of the current formulation. Finally, it is worth mentioning that the assumed stress filed satisfies equilibrium and all the boundary and continuity conditions and here, the principle of minimum complementary energy is implemented to get the effective stiffness matrix. Therefore, the current model is the most complete and versatile variational model developed so far based upon the single fundamental assumption that the in-plane stresses in each ply element are independent from the through-thickness direction.

2. Admissible stress field construction

Consider a symmetric multilayered laminate including 2 N perfectly bonded layers, which can have any combination of orientations while the symmetry about the mid-plane of the laminate is preserved. As laminate symmetry is assumed, it is better to consider only the upper set of N layers as shown in Fig. 1. A global set of rectangular Cartesian coordinates is chosen having the origin at the center of the laminate as shown in Fig. 1. The x-direction defines the longitudinal or axial direction, the y-direction defines the in-plane transverse direction and the z-direction defines the direction through the thickness. The locations of the N-1 interfaces of the first half of the laminate ($z > 0$) are specified by $z = z_i$; $i = 1, 2 \ldots N-1$. The mid-plane of the laminate is specified by $z = z_0 = 0$ and the external surface is demonstrated by $z = z_N = h$, where $2h$ is the total thickness of the laminate. The thickness of the ith layer is denoted by $h_i = z_i - z_{i-1}$. The orientation of the ith layer is specified by the angle θ_i (measured clockwise) between the x-axis and the fiber direction of this layer. The laminate must be such that the orientation of fibers in at least one set of plies is aligned in y-direction. This assumption is not a limitation because general in-plane loading conditions are considered so that if cracks form in another single orientation, the laminate can rotate so that the crack planes are parallel to the y-axis and the applied stresses transform to appropriate values for the new orientation. The stress and strain components and also material properties associated with the ith layer are denoted by a superscript or subscript i. Some layers might have similar properties, and therefore be modeled perfectly through the thickness variations in the stress fields. We assume that the laminate can be infinitely extended in both x and y directions (see Fig. 1), and consecutively the effect of the edges is neglected.

The laminate is subject to external uniform membrane loads of N_{xx}, N_{yy}, and N_{xy} in the coordinate system of xyz associated with cracks. In an un-cracked laminate, the only nonzero components of the stress tensor defined in the coordinate system associated with cracks are $\sigma_{xx}^{0(i)}, \sigma_{yy}^{0(i)}, \sigma_{xy}^{0(i)}$, where the superscript 0 denotes the undamaged state and the superscript (i), $i = 1,2, \ldots, N$, denotes the number of the layer. The stresses are spatially uniform within each layer and linear functions of the applied loads of N_{xx}, N_{yy}, and N_{xy}.

It is assumed that the ply crack distribution in damaged 90° plies is uniform, having a separation $2a$, and the cracks in each damaged 90° ply of the laminate are in the same plane. The cracked laminate can be seen as a sequence of laminate fragments, bounded by pairs of adjacent cracks (see Fig. 1). Further, it will be shown that each fragment has the same 'admissible' traction boundary conditions in the crack planes and hence can be treated separately. Fig. 2 indicates a fragment of length $2a$, which would be served as an elementary cell for constructing an admissible stress field. This fragment is enclosed by the mid-surface and top-surface of the laminate, the surfaces of two consecutive cracks, and a unit length along the parallel cracks. The origin of the coordinate system is located in the mid length of the fragment as shown in Fig. 2. The geometry of the fragment is then symmetrical with respect to the xy and yz planes.

Following the approach developed by Hashin (1985), the stresses in the cracked material are represented as a superposition of the stresses in the un-cracked material and some yet unknown perturbation stresses caused by the presence of the cracks.

$$\tilde{\sigma}_{mn}^{(i)}(X) = \sigma_{mn}^{0(i)} + \sigma_{mn}^{(i)}(X) \tag{2.1}$$