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Dynamic behaviour of a metamaterial system with negative mass and modulus

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ABSTRACT

Different models of metamaterials have been developed to generate negative mass and/or negative modulus. The resulting mass and modulus in existing works, however, cannot be independently controlled. The current study presents a new representative cell of elastic metamaterials in an effort to provide a comprehensive model for generating negative mass and/or negative modulus. The current model consists of a series of properly arranged rigid bodies and linear springs. By introducing both translational and rotational motions in the representative cell, negative mass and negative modulus can be obtained in a controlled manner. The mechanisms and conditions under which negative mass and/or negative modulus can be achieved are studied in detail. Numerical examples indicate that by varying the design of the representative cell, different properties of the material system can be reliably generated, i.e., double negative (mass and modulus) or single negative (mass or modulus). The dynamic behaviour of the developed material system under different loading frequencies is evaluated and the longitudinal elastic wave propagation in such metamaterials is studied.

1. Introduction

Metamaterials are typically artificial media designed to achieve unusual properties, which are not commonly seen in nature. Metamaterials have recently received significant attention from the research community (Huang and Sun, 2011; Fang et al., 2006) because of their potentials in designing new engineering structures with advantageous properties. Although under static loads typical structural materials have positive mass and elastic modulus, apparent negative mass and/or modulus can be observed when dynamic loading of a certain frequency is applied to properly structured material systems, forming acoustic/elastic metamaterials. The development of acoustic/elastic metamaterials will potentially enable the design of structures with unusual and attractive features, such as acoustic cloak for waves, noise elimination, and vibration reduction. Depending on the design of the material systems, the metamaterials can be single-negative (mass or modulus), or double negative (mass and modulus).

Significant progress in electromagnetic metamaterials has been achieved, from the earlier theoretical work on possible negative electric permittivity and magnetic permeability (Veselago, 1968) to the study of left-handed electromagnetic metamaterials and the design of lens with negative indexes of refraction (Smith et al., 2000; Pendry, 2000; Shelby et al., 2001). As the mechanical counterpart, the study of acoustic/elastic metamaterials has also received significant attention in recent years. By embedding heavy spheres coated with soft silicone in epoxy, apparent negative mass is observed at certain loading frequencies (Liu et al., 2000), which introduced the general concept of using local mechanical resonance to develop elastic metamaterials. The existence of negative mass density of acoustic metamaterials, formed by distributed particles in fluids, has also been observed numerically (Larabi et al., 2007; Mei et al., 2006). It is now well understood that negative mass can be generated for acoustic/elastic metamaterials using properly designed local mechanical resonators (Liu et al., 2005; Milton and Willis, 2007; Yao et al., 2008; Lee et al., 2009a).

Negative modulus has been observed in typical acoustic/elastic metamaterials. For a one dimensional acoustic metamaterial developed by using periodic Helmholtz resonators (cavities) (Fang et al., 2006), the experimental results show a negative group velocity, indicating a negative bulk modulus, similar to the phenomena presented in Lee et al. (2009b). A one dimensional elastic metamaterial consisting of periodic spring-mass cells also shows a negative modulus under certain frequencies (Zhao et al., 2012). A two-dimensional model of elastic metamaterials has been developed based on a spring-mass system with local resonators, which shows a frequency-dependent effective stiffness, being positive or negative depending on the frequency (Huang and Sun, 2011). A two-dimensional model of elastic metamaterials using a chiral structure has also been developed (Liu et al., 2011), from which negative mass and negative modulus can be generated

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simultaneously at certain frequencies. In this model the rotational motion is included to induce negative modulus. In a recent study (Bigoni et al., 2013), a similar concept is implemented in a two-dimensional model of elastic metamaterials with the unit cell of the metamaterial being formed by a central circular inclusion connected to the matrix by several inclined supporting beams (structured interface). From this material model negative group velocities can be observed at certain frequencies. Review of the recent development in the study of acoustic/elastic metamaterials can be found in references (Zhao et al., 2012; Bigoni et al., 2013; Fok et al., 2008). Typical applications of acoustic/elastic metamaterials include cloaking of acoustic/elastic waves and super resolution imaging (Zhao et al., 2012; Zhang et al., 2011; Parnell et al., 2013). In these applications a major issue is how to precisely control the effective material properties, both the mass and the modulus. But in the existing works discussed above negative modulus and negative mass cannot be controlled independently.

In the current study the generation of controllable negative mass and modulus is studied theoretically based on a new representative cell consisting of a series of springs and rigid bodies with both translational and rotational motions. The results indicate that the translational motion of the rigid bodies contribute directly to the effective mass and the rotational motion of them controls the effective modulus. By adjusting the geometry and material properties of the system, double negative metamaterials (negative mass and modulus) can be reliably generated. The resulting wave propagation and dispersion property are also discussed in detail.

2. Formulation of the problem

The problem considered is the harmonic mechanical response of a one-dimensional metamaterial system, shown in Fig. 1. The system is formed by a periodic structure with its representative cell consisting of two central rigid bodies, two end rigid bodies and linear elastic springs attached to the bodies, as shown in Fig. 1(b). The detailed description of the two central bodies are given in Fig. 1(c) and (d), which are circular in shape and can rotate freely around a common axis at the centre. In addition to the central springs attached to the axis, two springs are wrapped around each central rigid body at a radius of R, as shown in Fig. 1(b)–(d). The two central rigid bodies are under general plane motion and have the same mass and moment of inertia about their mass centres (the centre), m and I. When assembled, the six springs in Fig. 1(b) have elastic constants $2k$ and k', respectively, as identified in the figure, and are attached to two end rigid bodies of mass M. The length of the cell is assumed to be L, which is determined by the length of the springs and forms the characteristic length of the cell. The current arrangement of the cell ensures that the major geometries of the problem will remain unchanged during the motion. The two central rigid bodies will rotate in opposite directions and will eliminate possible rotating motion of the end rigid bodies. It should also be mentioned that the two central rigid bodies can be arranged in such a way that the cell will be symmetrical in the direction perpendicular to the plane considered, by sandwiching one body in the middle of the other one in the thickness direction, for example. The overall motion of the cell is then limited only to the longitudinal direction and the transverse motion of the system will be ignored. In the current model the free vibration of the system and the damping effect have been ignored.

It should also be mentioned that in the current model shown in Fig. 1 the springs will be under both tension and compression during the general harmonic motion. In a real physical system, the linear central springs can be designed to handle compression, but for springs wrapping around the central rigid bodies compression may result in buckling. In the current model, each spring wrapping around the central rigid bodies is actually two different springs with opposite directions. For example, the top spring in Fig. 1(c) represents a spring wrapping clockwise and attaching to the right rigid body and a spring wrapping counterclockwise and attaching to the left rigid body. Spring constant k' in the model is the difference between the constants of the two springs. The central springs are initially compressed and then assembled into the system. When balanced, the central springs will be under compression, but the springs wrapping around the central rigid body will be under tension, forming initial tension in the springs. By properly selecting the spring constants and the initial compression of the central springs, buckling of springs can be minimized by keeping the real wrapping spring mostly under tension during the harmonic motion. The motion of the physical system is described by the current model where k' and k represent the effective spring constants. With this in mind, in the current model for all springs the spring constants are assumed to be the same under tension and compression.

For the harmonic response of the system of frequency ω, all field variables will be in the general form of $\mathbf{A} = Ae^{-i\omega t}$ with A being the magnitude of either the displacement or the force. For convenience, in the following discussion the common term $e^{-i\omega t}$ will be omitted and only the amplitudes of these variables will be considered.

2.1. Effective modulus and effective mass

The response of the representative cell is governed by the displacements of the end masses and the forces acting on them. The displacements and forces at the two ends of the cell are denoted by u_n and u_{n+1}, and F_n and F_{n+1}, respectively, as shown in Fig. 1. The motion of the central masses is governed by both the translational displacement u_m at the centre, and the rotational displacement θ_m. By analyzing the dynamic response of the cell, u_m and θ_m can be determined as,

$$u_m = \frac{1}{1-\omega^2/\omega_1^2}\frac{u_{n+1}+u_n}{2}, \tag{1}$$

$$\theta_m = \frac{1}{1-\omega^2/\omega_0^2}\frac{u_{n+1}-u_n}{2R}, \tag{2}$$

where ω_1 and ω_0 are the natural frequencies of the central rigid body, given by

$$\omega_1^2 = \frac{2(k'+k)}{m}, \quad \omega_0^2 = \frac{2R^2k'}{G^2m}, \tag{3}$$

with R being the transverse distance between two neighbouring springs as illustrated in Fig. 1 and G being the radius of gyration of the central rigid body, i.e., $I = mG^2$. Eqs. (1) and (2) show clearly the resonance behaviour of both linear and angular motions, which will affect the apparent mass and modulus of the representative cell. The dynamic behaviour of the representative cell can be determined by using Eqs. (1) and (2) and solving the current dynamic problem. The solution can be represented in terms of forces and displacements at the ends of the cell as

$$\frac{1}{2}(F_{n+1}+F_n) = \left(k - \frac{M}{2}\omega^2 + \frac{k'}{1-\omega_0^2/\omega^2}\right)(u_{n+1}-u_n), \tag{4}$$

$$F_{n+1}-F_n = -\omega^2\left(2M + \frac{2m}{1-\omega^2/\omega_1^2}\right)\frac{u_{n+1}+u_n}{2}. \tag{5}$$

Eqs. (4) and (5) establish the relation between the average force applied to the cell and its deformation, and the relation between the net force applied to the cell and the average acceleration of it,