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Interval, ellipsoidal, and super-ellipsoidal calculi for experimental and theoretical treatment of uncertainty: Which one ought to be preferred?

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ABSTRACT

In this study we compare three calculi listed in the title for analysis of structures involving uncertainty. The main idea is based on the consideration that the maximum structural response predicted by the preferred theory ought to be minimal, and the minimum structural response predicted by the preferred theory ought to be maximal, to constitute a lower overestimation. We present analytic results that allow one to calculate the structural response via the interval, ellipsoidal or super-ellipsoidal calculus. We provide several examples of truss structures and illustrate that in different situations, depending on the available data, one of these calculi ought to be preferred. Conclusion is made on the preferable approach to be the super-ellipsoidal calculus.

1. Introduction

The "supermarket" of uncertainty analysis is quite large. Pate-Cornell (1996) lists six possible alternatives. Elishakoff (1990) describes three possible major classes in approaching uncertainty. There are: (a) probabilistic analysis involving probability densities and probability distributions of considered random variables, (b) fuzzy-sets based analysis that is based on the notion of the membership function, and (c) anti-optimization, or seeking for worst-case scenarios, and associated optimization to reduce the effect of the worst-case consequences. The latter technique is discussed in detail in the monographs by Hlavaček et al. (2004) and Elishakoff and Ohsaki (2010).

The central point of incorporating available, though a scarce, data into the analysis is discussed in the paper by Wang et al. (2008). In their study the authors advocate the idea that the choice of the uncertainty analysis must be determined by the experimental data. Specifically, anti-optimization analysis may involve either interval analysis as in the books by Moore (1966) and by Hansen and Walster (2004) or ellipsoidal analysis as in the monographs by Schweppe (1973), by Chernousko (1980, 1994) and by Ben-Haim and Elishakoff (1990). The natural question arises: "Which one, interval analysis or ellipsoidal analysis, ought to be preferred to its counterpart?" Wang et al. (2008) advocated the idea that the answer to the question depends on the experimental data; specifically one should prefer the analysis that produces least value for the maximum response so as to avoid overdesign.

In the recent article, Elishakoff and Bekel (2013) introduced new anti-optimization modeling based on super-ellipsoids (see Lame, 1818; Gardner, 1977; Sokolov, 2001), and derived analytical results for the maximum response of some structures involving uncertainty. This paper is a generalization of the two previous studies, namely papers by Wang et al. (2008) and Elishakoff and Bekel (2013). It proposes taking experimental data into account, bounding these data with an appropriate geometrical figure for two-dimensional uncertainty, namely a rectangle, ellipse or super-ellipse, and calculating the extreme displacements of the studied structure. We subscribe the philosophy articulated by Oden et al. (2010): "… theory and observation – the fundamental pillars of science – can be cast as mathematical models: mathematical constructs that describes system and represent acknowledge of the system in a usable form". Thus, this study combines experimental and analytical approaches. Conclusions are made as to the preference of the method that ought to be utilized. For other applications of super ellipsoids in applied mechanics the reader can consult with papers by Wang et al. (1994) and Ceribasi and Altay (2009).

2. Analytic results

In this study we concentrate on structures subjected to two independent uncertain loads; it is assumed that the experimental data describing their uncertainty is provided. Three candidate

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figures that are utilized to bound these data are two-dimensional. In addition, we consider the case when the expression of the response of the structure is a linear equation. Indeed, if X denotes the uncertain load applied on the node i, whereas Y denotes the uncertain load applied on the node j, the expression of the displacement u of the generic node k is expressible as follows

$$u = pX + qY \tag{1}$$

with p and q being the deterministic coefficients representing the elements occurring in the kth line and the ith and jth columns of the inverse of the stiffness matrix. Note that in the two-dimensional uncertainty case under consideration may involve additional, though determination forces which would lead to appropriate modification of Eq. (1).

Due to the linearity of the expression of the response of the structure in Eq. (1) and the Kelly–Weiss theorem in the paper by Ben-Haim and Elishakoff (1990), the extreme values of the displacements will necessarily be on the boundary of the bounding figure. A different method for each kind of bounding figure will be used to determine the extreme values of a given displacement. We will consider hereinafter three possible geometric figures that can be selected to bound the available data.

2.1. Determination of the extreme values for a given data enclosed by a rectangle

If one postulates the experimental data on (X, Y) to be represented by a rectangle, the extreme values of the response in Eq. (1) will be found by conducting a comparison between the values attained by the response function at the four vertices of the rectangle. Indeed, it appears inadvisable to study the response evaluated at other points inside the rectangle because of the linearity of the expression of the response. Namely, for the specified value u^* of the response in Eq. (1), the latter equation represents a line in the coordinate system OXY. Different values of $u = u^*$ correspond to different, but parallel, lines since the slope of the line (1) is independent of the value of u^* as depicted in Fig. 1. We have to establish the direction in which the value attained by u^* is increasing. Then the maximum value of u^* is attained by either by the line (1) or by the line (2), depending on the direction of increase of the response. If the line (1) corresponds to the maximum response, the line (2) is associated with the minimum one, and *vice versa*. Line (3) in Fig. 1 corresponds to neither maximum nor minimum.

In the case that the line (1) with $u = u_1^*$ turns out to be parallel to none of the sides (as it occurs in Fig. 1), two of the four vertices will lead to the extreme responses. Hereinafter, the points leading to the extreme responses are circled.

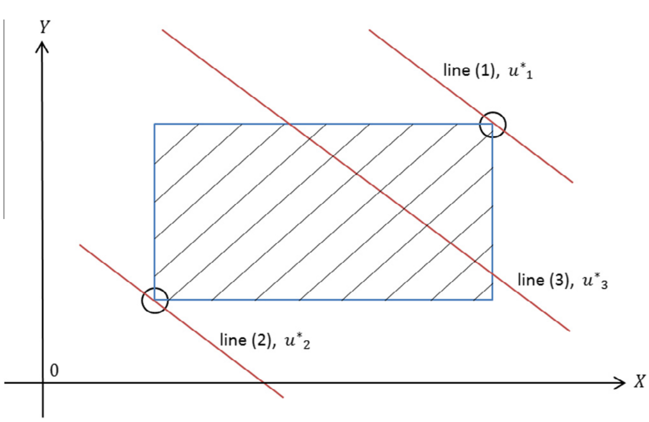

Fig. 1. Lines representing a constant value of the response are parallel to none of the sides of the rectangle (shared area contains experimental data).

If the line representing the constant response of the structure in Eq. (1) with $u = u^*$ is parallel to two sides of the rectangle (like in the Fig. 2), all the points of that side will lead to the same response. In such a special case, two of the four vertices will lead to the maximum response and the two other vertices will lead to the minimum response.

Hence, one just needs to compare the values provided by the four vertices to determine the extreme values of the response of the structure.

2.2. Determination of the extreme values for a given data enclosed by an ellipse

Let us now consider the case when the center of the ellipse is located at the point C (C_x, C_y); the ellipse is inclined by an angle α, as shown in Fig. 3. The equation of the sought displacement u is again given by the Eq. (1).

In a first step, we consider the ellipse in its local coordinate system which is centered at the origin 0, with semi-axes being along with $0X$ and $0Y$ axes, respectively (Fig. 4). The semi-major axis of this ellipse is denoted by a, whereas the semi-minor axis is designated by b.

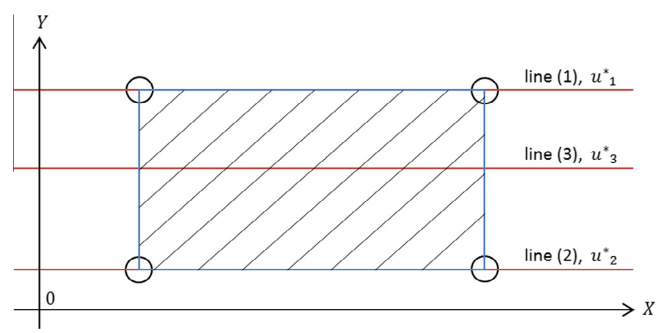

Fig. 2. Lines representing a constant value of the response are parallel to two sides of the rectangle (shared area contains experimental data).

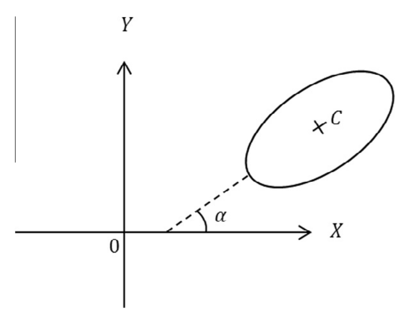

Fig. 3. Ellipse centered on a given point C and inclined by an angle α.

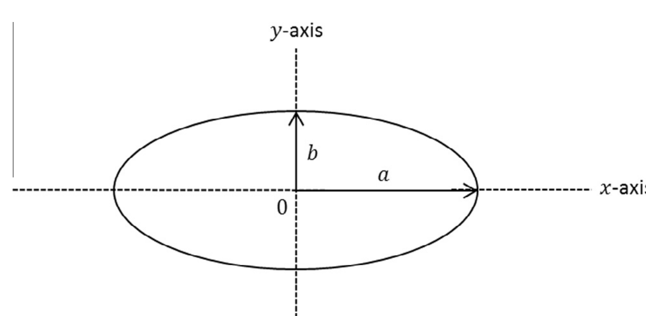

Fig. 4. Ellipse in its local coordinate system.