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Strain rate dependency of uniaxial tensile strength in Gosford sandstone by the Distinct Lattice Spring Model with X-ray micro CT

Gao-Feng Zhao [a,*], Adrian R. Russell [a], Xiaobao Zhao [b], Nasser Khalili [a]

[a] Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, The University of New South Wales, Sydney, NSW 2052, Australia
[b] School of Earth Sciences and Engineering, Nanjing University, Nanjing 210093, China

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ABSTRACT

Mechanisms causing strain rate dependency of the uniaxial tensile strength of Gosford sandstone are studied using the Distinct Lattice Spring Model (DLSM). The DLSM is built to have a microstructure which resembles aspects of the microstructure in a sample of the sandstone observed through 5 μm resolution X-ray micro CT scanning. Numerical dynamic uniaxial tensile tests on the sandstone are performed using both X-ray micro CT based and homogenous particle models. The results indicate that there is an only negligible strength increase with increasing strain rate for the homogenous particle model. However, a significant strength increase is observed with increasing strain rate for the X-ray micro CT based particle model. Therefore, it must be the microstructure that causes a strain rate dependency. Moreover, the influence of viscosity and rate dependency of springs are also studied. Results reveal that the rate dependency of the springs rather than their viscosity is also a main cause of the rate dependency.

1. Introduction

The dynamic failure of rock is a highly significant and timely problem in civil, mining, petroleum and geothermal engineering disciplines. It relates to economy and safety of structures built in or on rocks and is the key to solving engineering problems involving dynamic loading conditions. Brittle materials like rock, when viewed at a microscopic level, are discontinuous, heterogeneous and contain random flaws and cracks. The failure of these materials depends mainly on the size and spatial distributions of the flaws and cracks (Cox et al., 2005). A rigorous scientific approach to model these materials must consider the microstructure. However, most approaches have adopted conventional continuum mechanics, in which the constitutive laws used in describing the rock are derived from macro scale observations, without consideration of microstructure. Experimental studies often focus on the influence of loading rate on macro-mechanical properties, including deformation modulus, compressive strength, tensile strength and fracture toughness (e.g. Masuda et al., 1987; Li et al., 1999; Zhao, 2000; Yan and Gao, 2006; Shiro et al., 2008; Asprone et al., 2009; Cadoni, 2010; Dai et al., 2010). These studies provide at best a phenomenological description of dynamic effects in rock behavior. They are unable to consider microstructure, thus cannot explain fundamental mechanisms linking strain rate dependency and microstructure to fracturing and failure. There have also been several theoretical studies, which attempt to link loading rate to fracturing and failure. These are based on heat activation theory (Kumar, 1968), spring-dashpot models (Chong and Boresi, 1990; Chong et al., 1980), the sliding crack model (Li et al., 2000), and inertial effects (Li et al., 2009). These do offer a macroscopic interpretation of some rate dependent behaviors of rock materials, but still cannot explicitly describe the dynamic fracturing and failure processes, especially at the rock grain level (ranging in size from 10^{-3} m to 10^{-4} m).

With a focus on the microstructure, few researchers have used X-ray computed tomography (X-ray micro CT) and scanning electron microscopy (SEM) to study the micro fracturing (e.g. Ichikawa et al., 2001; Wang et al., 2005; Sufian and Russell, 2013). However, X-ray micro CT and SEM are only applicable under quasi-static loading. With the development of modern computer science, numerical methods provide a powerful alternative for micromechanical studies of rock and simulation of the dynamic fracture processes. Notable studies which have focused on the micro dynamic failure of rock materials include Du et al. (1989), di Prisco and Mazars (1996), Tang and Kaiser (1998), Zhu and Tang (2006), Zhou and Hao (2008) and Ma et al. (2011). In order for any numerical model to realistically describe rock dynamic fracture and failure, it should be able to account for the complex geometry at the micro scale. To tackle this problem, digital image data (both 2D and 3D) have been incorporated to different numerical methods, e.g. Finite Element Method (FEM) (Lengsfeld et al., 1998; Yue et al., 2003; Liu et al., 2004; Zhu et al., 2006; Dai, 2011), Finite Difference Method (FDM) (Chen et al., 2004, 2007), Discrete

* Corresponding author. Tel.: +61 2 9385 6139; fax: +61 2 9385 5022.
E-mail address: gaofeng.zhao@unsw.edu.au (G.-F. Zhao).

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Element Model (DEM) (Zeghal and Lowery, 2002), Lattice Boltzmann Method (LBM) (Bai et al., 2010; Degruyter et al., 2010), Moving Particle Semi-implicit method (MPS) (Ovaysi and Piri, 2010), Network modeling (Blunt et al., 2013), and lattice beam models (Van Mier, 1991; Schlangen and Van Mier, 1992; Asai et al., 2003; Schlangen, 2008). The lattice beam model solved the Poisson's limitation of the classical Lattice Spring Model (LSM), which has been used to study the well-known hand-shake crack pattern related to the softening of concrete (Van Mier, 1991). The digital image files can be obtained from a range of devices, for instance, the digital camera (e.g. Chen et al., 2004, 2007; Yue et al., 2003; Zhu et al., 2006), SEM (e.g. Schlangen and Garboczi, 1997), the Polarizing microscope (e.g. Liu et al., 2004), and X-ray micro CT (e.g. Lengsfeld et al., 1998; Schlangen, 2008). Among them, the X-ray micro CT is particularly attractive due to its non-destructive detection of 3D information.

In this work, the mechanism of strain rate dependency of uniaxial tensile strength in Gosford sandstone (part of the main Hawkesbury sandstone geological unit across the Sydney Basin) is studied using the Distinct Lattice Spring Model (DLSM) (Zhao, 2010; Zhao et al., 2011) with X-ray micro CT. Numerical dynamic uniaxial tensile tests on a virtual sample of sandstone are performed. The microstructure of the virtual sample replicates aspects of the 3D microstructure obtained from X-ray micro CT. This replication is possible using a technique for mapping the X-ray micro CT information onto the lattice used with the DLSM. The computational model is calibrated against a quasi-static uniaxial unconfined compressive test conducted on the sample from which the X-ray micro CT data was gathered. Numerical tests are conducted in which tensile failure occurs under different applied strain rates. The influences of microstructure, particle size, viscosity and rate dependency of springs are studied. Results reveal that the microstructure and rate dependency of the springs rather than the particle size and viscosity of the springs are main causes of the rate dependency.

2. The Distinct Lattice Spring Model

In the DLSM, solid material is represented by a particle model whose mechanical response is described by the corresponding lattice structure. In the model, two particles are linked through a spring bond whenever the gap between them is smaller than a prescribed threshold value (see Fig. 1a). Different particle assemblies and threshold values will produce different lattice structures (see Fig. 1(b) and (c)). DLSM can model both short- and long-range interactions through a bond that is made up from one normal spring and one multi-body shear spring (see Fig. 1a).

For the multi-body shear spring the relative shear displacement is obtained using a local strain of a particle cluster rather than the displacements of two linked particles. The shear deformation is calculated as (Zhao et al., 2011)

$$\hat{\mathbf{u}}_{ij}^{s} = [\boldsymbol{\varepsilon}]_{bond} \cdot \mathbf{n}l - (([\boldsymbol{\varepsilon}]_{bond} \cdot \mathbf{n}l) \cdot \mathbf{n})\mathbf{n} \tag{1}$$

where l is the initial bond length, $[\boldsymbol{\varepsilon}]_{bond}$ is the bond local strain which is evaluated as the average of the two linked particles, $\mathbf{n}$ is the normal vector of the spring bond. This approach has been proven to be able to keep rotational invariance and to solve the Poisson's ratio limitation in the conventional lattice spring models. The local strain technique also allows the DLSM to use only half the degree of freedoms compared with the Discrete Element Model (DEM).

Fig. 2 shows the failure criterion used in the DLSM. The bond is removed from calculation when the normal or shear deformation of the bond exceeds the prescribed value. After the bond is broken, if two particles tend to move towards each other, overlap is prevented by formation of a contact between the particles. New contacts can be formed at any time during the model deformation. Since the DLSM has only two spring parameters (spring normal and shear stiffnesses, k_n and k_s) and two failure parameters (spring deformations at which tensile and shear failure occur, u_t^* and u_s^*), it is a simple tool to observe and study the microstructure influence on the mechanical response of materials.

In the DLSM, broken bonds and new contacts are detected for given particle displacements from the previous calculation step. Then, spring and contact forces are calculated as per the constitutive law (Fig. 2). The acceleration of each particle is calculated from Newton's second law as

$$\ddot{\mathbf{u}}^{(t)} = \frac{\mathbf{F}^{(t)}}{m_p} \tag{2}$$

where $\mathbf{F}^{(t)}$ is the sum of forces acting on the particle, m_p is the particle mass.

Then, the particle velocity is updated as

$$\dot{\mathbf{u}}^{(t+\frac{\Delta t}{2})} = \dot{\mathbf{u}}^{(t-\frac{\Delta t}{2})} + \ddot{\mathbf{u}}^{(t)}\Delta t \tag{3}$$

where Δt is the time step.

Finally, the new displacement of the particle is

$$\mathbf{u}^{(t+\Delta t)} = \mathbf{u}^{(t)} + \dot{\mathbf{u}}^{(t+\frac{\Delta t}{2})}\Delta t \tag{4}$$

It was proven that by using Newton's second law with a lattice structure, the classical elastic and elastic dynamic problems can be successfully solved by the DLSM and the results are in good agreement with corresponding analytical solutions (Zhao, 2010; Zhao et al., 2011; Zhu et al., 2011).

Another distinct feature of the DLSM is that the input elastic parameters are the macro material constants, i.e. the Young's modulus E and the Poisson ratio v. Spring stiffness parameters are automatically calculated using following equations (Zhao, 2010; Zhao et al., 2011):

$$k_n = \frac{3E}{2\alpha^{3D}(1-2v)} \tag{5}$$

$$k_s = \frac{3(1-4v)E}{2\alpha^{3D}(1+v)(1-2v)} \tag{6}$$

where E is the Young's modulus, and v is the corresponding Poisson's ratio. The α^{3D} is the microstructure geometry coefficient of the lattice model.

$$\alpha^{3D} = \frac{\sum l_i^2}{V} \tag{7}$$

where l_i is the original length of the ith bond, V is the volume of the represented geometry model. The underlying principle to link the micro and macro parameters is that the total strain energy of the lattice structure is equivalent to that of the continuum model (see Fig. 3). Details of derivation and verification of DLSM can be found in Zhao (2010) and Zhao et al. (2011).

3. X-ray micro CT test

The X-ray micro CT data used, from Sufian and Russell (2013), is for Gosford sandstone, which is a medium grained (0.2–0.3 mm), poorly cemented, immature quartz sandstone. It contains 20–30% feldspar and clay minerals, with serrate connection between quartz grains, and has an average density of 2265.5 kg/m^3 (Ord et al., 1991). Sufian and Russell (2013) took images of the central 5.24 mm portion of a 5 mm diameter and 19.2 mm long core (see Fig. 4). The X-ray micro CT scanning was conducted at the Australian Centre for Microscopy and Microanalysis (ACMM) using the