International Journal of Solids and Structures 51 (2014) 2878–2892

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Development of a distributed dislocation dipole technique for the analysis of multiple straight, kinked and branched cracks in an elastic half-plane

N. Hallbäck *, M.W. Tofique

Department of Engineering and Physics, Karlstad University, 65188 Karlstad, Sweden

ARTICLE INFO

Article history:
Received 27 November 2013
Received in revised form 28 March 2014
Available online 24 April 2014

Keywords:
Cracks
Dislocation dipoles
Stress intensity factors
Singular integral equations

ABSTRACT

A distributed dislocation dipole technique for the analysis of multiple straight, kinked and branched cracks in an elastic half plane has been developed. The dipole density distribution is represented with a weighted Jacobi polynomial expansion where the weight function captures the asymptotic behaviour at each end of the crack. To allow for opening and sliding at crack kinking and branching the dipole density representation contains conditional extra terms which fulfills the asymptotic behaviour at each endpoint. Several test cases involving straight, kinked and branched cracks have been analysed, and the results suggest that the accuracy of the method is within 1% provided that Jacobi polynomial expansions up to at least the sixth order are used. Adopting even higher order Jacobi polynomials yields improved accuracy. The method is compared to a simplified procedure suggested in the literature where stress singularities associated with corners at kinking or branching are neglected in the representation for the dipole density distribution. The comparison suggests that both procedures work, but that the current procedure is superior, in as much as the same accuracy is reached using substantially lower order polynomial expansions.

1. Introduction

In many fatigue and fracture assessment cases, particularly early stages of fatigue, defects and cracks are small compared to other dimensions of the body. When fatigue is an issue it is also well known that initiation and incipient growth of short cracks dominates the total life of the structure. In such cases it is unnecessary to solve for the stress and strain state in the entire domain, except for a small neighbourhood adjacent to the crack or defect. Hence, the Finite Element Method (FEM) is not ideally suited for such situations. Due to the relatively low level of loading required to initiate and propagate cracks it is also evident that the body will mainly behave linear elastic. This is particularly so in case of Very High Cycle Fatigue (VHCF) (Bathias, 1999). Taking these facts into consideration, the problem is thus ideally suited to be treated by some Boundary Element Method (BEM) approach. However the presence of cracks inflicts some difficulties in traditional BEM (Cruse, 1988) based on infinite plane fundamental solutions for the displacements, since the upper and lower crack surfaces of a crack coincide, leading to identical equations for collocation points taken on opposite positions along the crack. The so called Dual Boundary Element Method (DBEM, see Portela et al., 1992; Mi, 1996), where Traction Boundary Integral Equations (BIE's) are used in combination with Displacement BIE's along the crack, has been used to circumvent this anomality, but there are other alternatives.

One alternative is the Distributed Dislocation Technique (DDT) where the derivative of the relative opening and sliding displacements of the crack surfaces are represented by distributions of dislocations (Bilby and Eshelby, 1968), or the Distributed Dislocation Dipole Technique (DDDT) where the opening and sliding displacements are represented by dislocation dipoles (Korsunsky and Hills, 1995). Both methods are described in detail in Hills et al. (1996). Yet another, but related technique, is the Displacement Discontinuity Method (DDM) originally developed by Crouch (1976). This technique is based on the stresses and displacements that results at a point due to constant displacement discontinuities over a finite length line segment in the body. This technique has been further improved by introducing line segments with higher order variations of the displacement discontinuities (Shou and Crouch,

* Corresponding author. Tel.: +46 54 700 21 15.
E-mail addresses: nils.hallback@kau.se (N. Hallbäck), muhammad-waqas.tofique@kau.se (M.W. Tofique).

http://dx.doi.org/10.1016/j.ijsolstr.2014.04.011
0020-7683/

1995). When applied to crack problems all of the abovementioned methods are usually adapted, by use of weight functions or by special interpolation formulas for the crack tip elements, to capture the square root singular behaviour of the stresses at the tip of a crack. By doing so, the analyses of stress intensity factors for straight cracks (single or multiple) are most often proven to be very efficient and accurate.

Of special interrest here is therefore the analysis of stress intensity factors in cases of kinked and/or branched cracks. If the kink or branch is disposed in such a way that an inward-bounded corner is formed in the material the stress state becomes singular, whereof the symmetric part possess the strongest singularity, while the asymmetric part exhibits a weaker singularity. Most often, however, the stress singularity associated with crack kinking is neglected in the formulation of kinked (or branched) crack problems. Zang and Gudmundson (1988) adopts the DDT for the part of the BIE's taken along the cracks and argue that the strongest stress singularity should be used to represent the asymptotic behaviour at a kink. In their benchmark analysis, however, they assume regular kink behaviour. Regular behaviour at crack kinking is also assumed in the BEM developed by Wang and Chau (1997), in the DDDT developed by Denda and Dong (1999) and in Marji and Dehghani (2010) who used a higher order DDM for the analyses of kinked cracks. At the other extreme the stress singularity at crack kinking is taken into consideration by assigning an oversevere, crack tip singularity, at the kink. This approach was taken in the DDT developed by Yingzhi and Hills (1990). A DDT accounting for both the asymmetric and symmetric singularity at crack kinking was presented by Burton and Phoenix (2000), and was further refined by Yavuz et al. (2006). Their method enables accurate determination of both the stress intensity factors as well as the strength of the singularities at crack kinking. In both studies the ableness of the method was demonstrated on single, or multiple, kinked or branched cracks in an infinite plane. Apart from edge cracks, branched cracks with singular behaviour at the branch were also excluded in their sample calculations.

While several investigators have been devoted to the development of the DDT to analyse kinked and branched cracks, less efforts have been made to apply the DDDT. A direct comparison between the two methods by Hills et al. (1996), on straight edge and interior cracks in a half plane, reveals that the DDDT requires less degrees of freedom in comparison to the DDT in order to obtain the same accuracy. This is particularly pronounced for edge cracks, since the DDDT does not inflict any artificial constraints on the behaviour at the crack mouth as opposed to the DDT. Hence convergence is accelerated with the DDDT in those cases.

Encouraged by these facts the DDDT is adopted in this development. The development is carried out for 2D, but the method is in principal applicable in 3D as well, albeit considerably more involved. The aim of this document is to present the method in a unified, but yet detailed and comprehensive, manner covering all possible situations within the scope of the title of the document. Possible contact between the crack surfaces is however disregarded, whence the method applies only for cracks that stay fully open. Care is taken to account for the singular behaviour of the stress field at crack kinking and branching, and to assess the accuracy and efficiency of the procedure as compared to the simplified approach where stress singularities at crack kinking/branching is neglected, such as in procedures developed by Denda and Dong (1999) among others. As proposed by Zang and Gudmundson (1988) the stress singularity at crack kinking is here assumed to be completely governed by the strongest singularity at the kink, which of course is an approximation to the exact nature of the singularity at a corner. The herein presented method could be extended to the analysis of finite two dimensional bodies by using techniques similar to that presented in Dai (2002).

1.1. Problem definition

Multiple surface or interior cracks close, or remote, to a free surface of an infinite half plane are considered. The cracks may either be straight or possess multiple kinks and/or branches. The number of straight crack segments, either in the form of straight cracks, or as a part of kinked and/or branched cracks, are denoted by N. The inclination angle of each straight crack segment referred to the positive x_1-axis is denoted by θ_i, where i denotes the segment number (see Fig. 1). The center point of each crack segment referred to the global (x_1, x_2) system is denoted by $(c_1^{(i)}, c_2^{(i)})$. This report focuses on the evaluation of the stress intensity factors at each crack tip. A brief description on how to evaluate the stress state at an arbitrary location in the body will also be given.

2. Method

The numerical procedure is based on the fact that the stress intensity factors due to a remotely applied stress field can be computed by applying equivalent stresses acting on the crack surfaces in an identical but otherwise unloaded body. This principle is known as Bueckner's principle (Bueckner, 1958) which states that the equivalent stresses are the opposite to the stresses that act along cutting planes (coincident to crack segments in the cracked body) in the equivalent uncracked body subjected to the remote stress field in question. The cracks will respond to the applied stresses by opening and sliding displacements between the opposing crack surfaces. These displacements are represented by unknown distributions of dislocation dipole densities along each crack segment. The unknown coefficients of the distributions are determined by requiring that the stress state along the crack segments should be fulfilled. The stress conditions are enforced at certain points (collocation points), which results in two coupled integral equations for each component of stress at each collocation point. In addition to the collocation equations there are also certain continuity equations that have to be fulfilled at crack kinking and branching. By choosing an appropriate number of collocation points, the collocation integral equations and the continuity constraints results in a linear equation system which could be solved to obtain the unknown dislocation dipole density distribution. Once the dislocation dipole density distribution is known the stress intensity factors, and the stress state at any point in the body, could be evaluated.

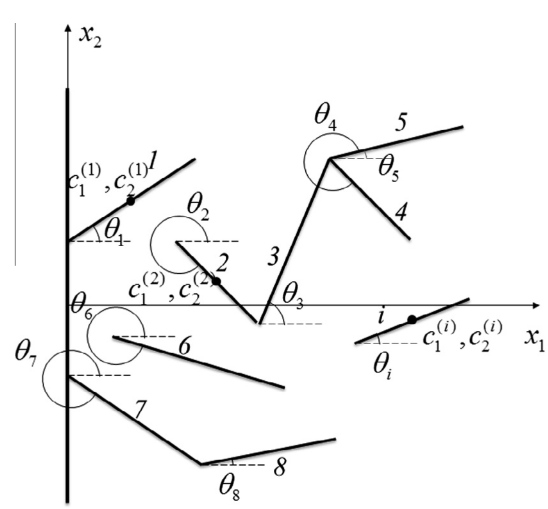

Fig. 1. Multiple straight, kinked and branched edge or internal cracks in a half plane.