International Journal of Solids and Structures 51 (2014) 2908–2919

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Analysis of a three-dimensional dissimilar material joint with one real singularity using a conservative integral

Chonlada Luangarpa *, Hideo Koguchi

Nagaoka University of Technology, 1603-1 Kamitomioka, Nagaoka, Niigata 940-2188, Japan

ARTICLE INFO

Article history:
Received 23 August 2013
Received in revised form 2 April 2014
Available online 2 May 2014

Keywords:
Three-dimensional joints
Stress singularity
Conservative integral
Intensity of singularity
Betti reciprocal theorem

ABSTRACT

In the present study, a conservative integral based on the Betti reciprocal principle is formulated to determine the intensity of singularity at a vertex of the interface in three-dimensional dissimilar material joints with one real singularity. Eigenanalysis formulated using a three-dimensional finite element method (FEM) is used to calculate the order of stress singularity, angular functions of displacements and stresses. Models with various element sizes and various integral areas are used to investigate the effect of the integration area on the accuracy of the results. The results are compared with those obtained from the boundary element method (BEM) using a curve-fitting technique to calculate the intensity of singularity. In addition, models of various lengths and various material combinations are used to investigate the stress singularity characteristics in three-dimensional dissimilar material joints. The results of the present study indicate that the conservative integral can be used to determine the intensity of singularity in three-dimensional bi-material joints. The accuracy of the results can be improved by mesh refinement. Finally, the relationships among the intensity of singularity, the order of stress singularity and the model geometry are discussed.

1. Introduction

Dissimilar material joints have singularities created by discontinuities in material properties across interfaces. A mismatch in the material properties of joints may lead to fracture and failure; thus, the investigation of stresses in dissimilar material joints is important. It is difficult to accurately calculate stress distributions near the edge of an interface in joints using the conventional finite element method (FEM). Special elements and methods have been developed to solve singular stress problems, and these methods can be classified into two groups: (1) methods based on the development of a special element, e.g., a hybrid element method (Tong et al., 1973), an enriched FEM (Benzley, 1974), and extended/generalized finite element methods (XFEM/GFEM) (Belytschko and Black, 1999; Fries and Belytschko, 2010) and (2) methods based on a post-process for calculating the intensity of singularity, e.g., a stress extrapolation (Munz and Yang, 1993) and a conservative integral (Banks-Sills and Sherer, 2002). The above examples are two-dimensional analyses, some of which are being developed for extension to three-dimensional analyses.

Singular stress fields are more complicated for three-dimensional dissimilar material joints than for two-dimensional dissimilar material joints because singular stresses are generated not only at vertices but also along the free edges of the interface. There have been many studies on the analysis of the order of singularity in three-dimensional cases. For example, Ghahremani (1991) used a numerical variational method and Pageau et al. (1995) and Koguchi and Muramoto (2000) used FEM eigenanalysis to analyze the order of singularity. However, there have been only a few studies on the analysis of the intensity of singularity. For example, Lee and Im (2003) used the two-state M-integral, Wisessint and Koguchi (2009) developed a 3D enriched FEM and Koguchi and Da Costa (2010) used the BEM with a curve-fitting technique to analyze the intensity of singularity.

Many studies have revealed that the conservative integrals, such as the J-integral, M-integral, and H-integral, are powerful methods by which to obtain intensities of singularities. Using these methods, special elements or very refined meshes around the singular point are not necessary. The H-integral method based on the Betti reciprocal has a simple form of the integral, which requires only the displacements and stresses (Ortiz et al., 2006). In particular, in three-dimensional analysis, variables of displacements and stresses cannot be calculated analytically. In addition, numerical methods may have unavoidable numerical errors. Therefore,

* Corresponding author. Tel.: +81 8056887875.
E-mail addresses: chonlada@stn.nagaokaut.ac.jp (C. Luangarpa), koguchi@mech.nagaokaut.ac.jp (H. Koguchi).

http://dx.doi.org/10.1016/j.ijsolstr.2014.04.018
0020-7683/

methods that require fewer variables or calculation steps may provide more accurate results. Thus, in the present study, the H-integral is developed for use in three-dimensional analysis.

The conservative integral based on the Betti reciprocal principle was first developed for calculating the stress intensity factors of notched homogeneous bodies (Stern et al., 1976; Sinclair et al., 1984; Carpenter, 1984). Carpenter and Byers (1987), Banks-Sills (1997), and Banks-Sills and Sherer (2002) later extended this method to solve for the intensity of singularity for two-dimensional dissimilar material joints. Many studies have investigated the intensity of singularity in three-dimensional dissimilar material joints using the conservative integral. For example, Meda et al. (1998) solved three-dimensional crack problems, Ortiz et al. (2006) used H-integral together with BEM to determine the stress intensity factor along three-dimensional crack fronts and Nomura et al. (2010) analyzed the interfacial corner between anisotropic bi-material under thermal stress. Moreover, Kuo and Hwu (2010) examined anisotropic material interface corner problems involving mechanical loading. Most of the previously developed three-dimensional formulations used for evaluating the intensity of singularity at interface corners or cracks assumed the integral area to be a cylindrical closed surface. Therefore, the previous formulations are difficult to extend when determining the intensity of singularity at a vertex of the interface. Furthermore, most of the previous studies in dissimilar material joints used the order of singularity and angular variables of displacements and stresses based on two-dimensional analysis, which implies that the plane strain condition was assumed in the analysis.

In the present analysis, the conservative integral is developed to analyze the intensity of singularity at the vertex of an interface in three-dimensional bi-materials with one real singularity. The eigenanalysis formulated by a three-dimensional finite element method (FEM) (Pageau et al., 1995, 1996; Pageau and Biggers, 1995) is used to calculate the order of stress singularity and variables of the displacements and stresses. In particular, most previous studies used a two-dimensional analysis to calculate the order of stress singularity and the angular functions. The primary difference between two-dimensional eigenanalysis and three-dimensional eigenanalysis is a complementary solution that is used in a conservative integral. For the conservative integral method, two sources of solutions, i.e., singular and complementary solutions, are needed. For two-dimensional analysis, $2-\lambda$ (where λ is the order of singularity, $0<\lambda<1$) is used as the complementary solution, and $3-\lambda$ is used for a three-dimensional analysis. Lee and Im (2003) reported $3-\lambda$ to be an eigenvalue for three-dimensional analysis. The solutions obtained from a conventional FE analysis together with the solutions deduced from FEM eigenanalysis are then used to calculate the intensity of singularity. To our knowledge, no study on the determination of the three-dimensional intensity of singularity using the H-integral has been conducted.

To investigate the effect of mesh refinement and the integral area on the accuracy of the results, models with various element sizes and integral areas are used. The obtained results are compared with BEM results obtained using a curve-fitting technique to calculate the intensity of singularity (Koguchi and Da Costa, 2010). Then, to investigate the effect of the model's geometry and the material properties on the singular stress field, models of various lengths and various material combinations are investigated.

The remainder of the present paper is organized as follows. Section 2 presents the analytical formula for the vertex in three-dimensional joints. The numerical analysis is explained in detail in Section 3, which is divided into four subsections. Section 3.1 presents the analytical model and boundary conditions. The order of the stress singularity and the angular functions obtained by FEM eigenanalysis are described in Section 3.2. Section 3.3 presents the results obtained from models of various element sizes and integral areas. Next, the relationships among the intensity of singularity, the model geometry, and the material properties are discussed in Section 3.4. Finally, conclusions are presented in Section 4.

2. Analytical formula

A conservative integral for a three-dimensional joint is developed using Betti's reciprocal principle as follows:

$$\int_S (T_i' u_i - T_i u_i')\,ds = 0. \tag{1}$$

For any contour S, T_i and T_i' are tractions, and u_i and u_i' are the displacements of the singular and complementary fields. This principle is extended to solve the three-dimensional bi-material model, as shown in Fig. 1. Eq. (1) is rewritten as an integral with respect to the closed area shown in Fig. 2 as follows:

$$\sum_{j=0}^{5} \int_{S_j} (T_i' u_i - T_i u_i')\,ds = 0. \tag{2}$$

The contour, S, is selected to be $S = S_0 + S_1 + S_2 + S_3 + S_4 + S_5$, where S_2, S_3, S_4 and S_5 are on free-surfaces; then, the traction is free on these surfaces such that

$$\int_{S_0} (T_i' u_i - T_i u_i')\,ds + \int_{S_1} (T_i' u_i - T_i u_i')\,ds = 0. \tag{3}$$

Modifying the form of traction to be $T_i = \sigma_{ij}\hat{n}_j$ yields

$$\int_{S_0} \left(\sigma_{ij}' u_i - \sigma_{ij} u_i'\right)\hat{n}_j\,ds + \int_{S_1} \left(\sigma_{ij}' u_i - \sigma_{ij} u_i'\right)\hat{n}_j\,ds = 0, \tag{4}$$

where $\hat{n}_j$ is the outward unit vector to the closed surface, S (see Fig. 3). Let $\hat{n}_j'$ be the unit vector in the direction opposite $\hat{n}_j$, such that

$$\int_{S_0} \left(\sigma_{ij}' u_i - \sigma_{ij} u_i'\right)\hat{n}_j'\,ds = \int_{S_1} \left(\sigma_{ij}' u_i - \sigma_{ij} u_i'\right)\hat{n}_j\,ds. \tag{5}$$

Eq. (5) indicates that the integral is area independent.

Finally, the H-integral at the vertex in three-dimensional dissimilar materials is defined as follows:

$$H = \int_{S_\Gamma} \left(\sigma_{ij}' u_i - \sigma_{ij} u_i'\right)\hat{n}_j\,ds, \tag{6}$$

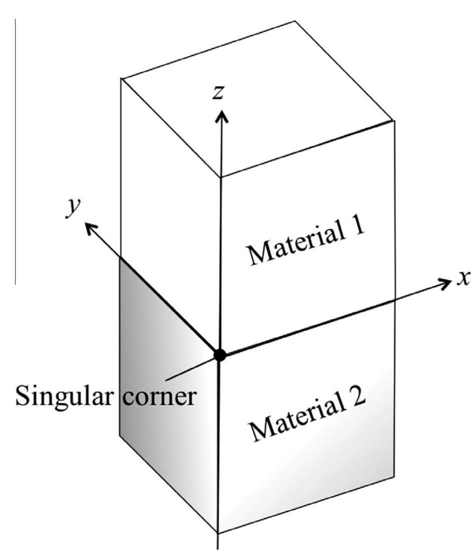

Fig. 1. Three-dimensional bi-material joint model.