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A novel method for residual strength prediction for sheets with multiple site damage: Methodology and experimental validation

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ABSTRACT

A unified method for solving the strip yield model for collinear cracks in finite and infinite sheet is proposed. The method is based on the weight function of a single crack. Two collinear cracks in finite and infinite sheets are used to apply and verify this method. The plastic zone size, crack opening displacement and stress distribution along the ligament between cracks obtained by using the present method are extensively compared with existing available results and finite element solutions, and very good agreements are observed. Combined with the Crack Tip Opening Angle (CTOA) criterion, the unified method is used to predict the crack growth behavior and residual strength for 2024-T3 aluminum alloy sheet with Multiple Site Damage (MSD). Thirty-two sheets with four types of MSD are designed and tested to verify this method. It is shown that the present method is able to predict various crack growth behaviors observed in experiment. The predicted residual strengths are within 9% of the corresponding test results. Compared to the elastic–plastic finite element method, the present method is much more efficient.

1. Introduction

Aircraft structures are designed to retain adequate structural integrity in the presence of major damage. The Aloha Airlines accident in 1988 resulted in much attention being paid to the Multiple Sites Damage (MSD) phenomenon of riveted lap joints in aircraft fuselages (Swift, 1994). MSD was characterized by the simultaneous presence of fatigue cracks in the same structural element, for example, fatigue cracking at multiple rivet locations in lap joints (FAA, 2010). Subsequently, procedures for maintaining the structural integrity of aging aircraft were established. However, there were still several civil and military aircraft failures reported due to the presence of Widespread Fatigue Damage (WFD) (FAA, 2010). In order to maintain the aircraft safety, in 2010, the Federal Aviation Administration (FAA) in US issued its newest rules to prevent catastrophic failure due to WFD throughout the operational life of certain existing transport category airplanes and all those to be certificated in the future (FAA, 2010). These incidents and the FAA requirements call for suitable methods to analyze the behaviors of multiple fatigue cracks in susceptible structural locations.

During the past two decades, various methodologies and fracture criterions had been developed to predict the residual strength of MSD structures (Cherry et al., 1997; Jeong and Brewer, 1995; Labeas et al., 2005; Ma et al., 1996; Moukawsher et al., 1996; Wang et al., 1997; Smith et al., 2001). For thin aluminum alloy sheet, large plastic zone was observed in the residual strength analysis. Consequently, the well-established linear elastic fracture mechanics criterions were shown to be inaccurate or unable to predict the residual strength and stable crack growth for structures with MSD. The elastic–plastic fracture parameter, Crack Tip Opening Angle/Displacement (CTOA/D), was widely used to predict the residual strengths for stiffened and unstiffened panels with or without MSD (Dawicke and Newman, 1998; Hsu et al., 2003; Mahmoud and Lease, 2004; Newman et al., 2003a,b; Seshadri et al., 2003). In the practical application, the CTOA criterion was usually combined with elastic–plastic finite element method or the strip yield model. Many researchers had used the CTOA criterion and elastic–plastic finite element method to predict the stable crack growth behaviors and residual strengths for various cracked configuration. Most of the predicted results were reported to be within 10% of the experimental data. However, when finite element method was involved, large computational and modeling demands as well as experiences were required. Since Dugdale (1960) strip yield model is relative simple to calculate the plastic zone size and crack opening displacement, it has been frequently combined with the CTOA criterion to predict the stable crack growth and residual strength (Deng and Hutchinson, 1996; Kuang and Chen, 2000; Nilsson and Hutchinson, 1994; Nilsson, 1999). And various methods were

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developed to solve the strip yield model for collinear cracks, for example, the complex stress function method (Collins and Cartwright, 2001), the Fredholm integral equation method (Nishimura, 2002). However, most existing analytical methods encounter difficulties especially in solution efficiency and in the ability of analyzing the effect of finite width on MSD, which cannot be ignored in the practical engineering analysis. Recently the weight function method (Wu and Carlsson, 1991) has been successfully used by the present writers to tackle the MSD problems (Wu and Xu, 2011; Xu et al., 2011; Xu and Wu, 2012).

In this paper, a simple and highly efficient analytical method based on the weight function of a single crack is proposed to analyze the strip yield models for collinear cracks, including the finite width effect. The strip yield models for two collinear cracks in infinite and finite sheets are taken as examples to demonstrate and verify the present method. The stress distributions along the ligament between cracks, the plastic zone sizes and crack opening displacements are determined. These results are extensively compared with those from previous studies and finite element results, and perfect agreements are observed. It is also observed that the present method is much simpler and more efficient than the other methods to analyze the strip yield model for collinear cracks. After the validation of the present method, it is combined with the CTOA criterion to predict the stable crack growth and residual strength for sheet with MSD subjected to tension loading. Thirty-two sheets made of 2024-T3 aluminum alloy with various MSD-patterns are designed and tested to verify the accuracy and efficiency of present method. It is shown that the present method is able to predict the stable crack growth behaviors well, and the predicted residual strengths are within 9% of the corresponding experimental results. Compared with elastic–plastic finite element method, the present method is much more efficient.

2. Residual strength prediction model

2.1. CTOA criterion based on strip yield model

The CTOA criterion combination with the strip yield model had been used by several researchers to predict the stable crack growth behavior (Deng and Hutchinson, 1996; Newman, 1986; Nilsson and Hutchinson, 1994; Nilsson, 1999). In the method, the crack growth was controlled by two parameters. One was the critical crack opening displacement δ_0 which was used to control the crack initiation; and the other was applied to characterize the stable crack growth behavior by a critical constant CTOA α_c.

A single crack configuration shown in Fig. 1 is taken as an example to describe the crack growth analysis criterion and process. The crack will be initiated when the crack tip opening displacement reaches the critical value δ_0, see Fig. 1(a). Subsequently, it will grow forward. Fig. 1(b) shows the crack opening profile when the crack has propagated a distance Δa. Plastic wake is left behind as the crack advances. The elastic deformation in the wake is much less than the plastic stretch. As a result, the height of the plastic wake can be approximated to be equal to the prior crack stretch. The further crack growth was controlled by the critical CTOA α_c. The crack opening angle was defined by the crack opening displacement at a small distance $d = 1$ mm behind the crack tip. When the crack tip opening angle reaches the critical value α_c, the crack will propagate a distance $d = 1$ mm. Consequently, the crack growth equation for the whole process is

$$\begin{cases} \delta(a_0, a_0) = \delta_0; & \text{crack initiation} \\ \delta(a, a-d) = 2d\tan(\alpha_c/2) + \delta_c(a-d, a-d); & \text{crack propagation} \end{cases} \quad (1)$$

where, $\delta(a, a-d)$ is the crack opening displacement at a distance d behind the crack tip, the first variable in the bracket is the current crack length, and the second is the x location. $\delta_c(a-d, a-d)$ is the plastic wake height, which is equal to the crack tip opening displacement. α_c is the critical crack opening angle.

Newman (1986) defined the right term in the second equation as crack tip opening displacement resistance curve V_R. Usually, the plastic wake increases with the crack growth, which results in the crack growth resistance. It was found (Newman, 1986) that the V_R curve was independent of the initial crack length, specimen width and geometrical type, but was a function of material and specimen thickness.

The criterion for a single crack was also adopted to predict the each crack growth for plates with MSD (Deng and Hutchinson, 1996; Galatolo and Nilsson, 2001; Nilsson and Hutchinson, 1994), see Fig. 2(a). The crack initiation criterion for sheets with MSD was

$$\delta(a_{0i}, a_{0i}) = \delta_0 \quad (2)$$

where a_{0i} is the ith crack length. And, the equations for propagation were

$$\begin{cases} \delta(a_i, a_i-d) = 2d\tan(\alpha_c/2) + \delta_c(a_i-d, a_i-d); & +x \text{ direction} \\ \delta(a_i, a_i+d) = 2d\tan(\alpha_c/2) + \delta_c(a_i+d, a_i+d); & -x \text{ direction} \end{cases} \quad (3)$$

Besides the crack initiation and propagation equation, a criterion for crack linkup (Fig. 2(b)) is needed to predict the crack growth behavior for sheets with MSD. Theoretically, the link-up criterion is the current ligament $l_i = 0$ mm. It is observed from experiment that the ligament will break immediately when the current ligament length l_i is small. Here, the equation for crack coalescence given in Galatolo and Nilsson (2001) is applied,

$$l_i \leqslant 2\text{ mm}; \quad i\text{th ligament fracture} \quad (4)$$

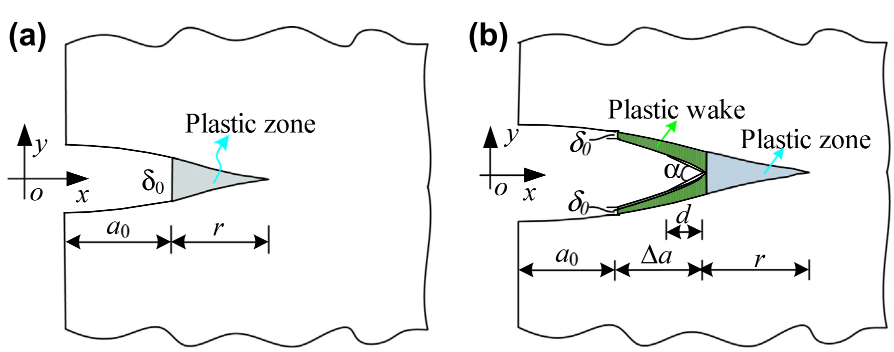

Fig. 1. Crack opening profile of modified Dugdale strip yield model at (a) initiation and (b) at propagation with definition of crack growth parameters α and δ_0.