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Mechanics of adhesive contact at the nanoscale: The effect of surface stress

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ABSTRACT

At small length scales, the adhesion and surface effect are of great significance, both of which play important roles in the contact between two elastic solids. In this study, the classical Johnson–Kendall–Roberts (JKR) adhesive contact theory is generalized to the nanoscale at which the surface effect is considered. The influence of the surface stress on the JKR adhesive contact is investigated by employing the non-classical Boussinesq fundamental solutions. It is found that, compared with the classical theory, the pull-off force increases while the critical contact radius decreases as a result of the surface effect. Numerical results show that a relative error of 10% can be introduced in the pull-off force when the indenter radius is less than 20 nm. A detailed theoretical analysis of this interesting phenomenon is presented based on dimensional analysis, and two scaling laws for the adhesive contact at the nanoscale are constructed. These two new scaling laws reveal that the pull-off force is relevant to the elastic properties of the bulk materials, which is different from the classical adhesive contact theory. The present work is promising for the engineering applications in micro-electro-mechanical systems (MEMS) and nano-intelligent devices.

1. Introduction

Tremendous progresses have been made in nanotechnology in recent decade because of its promising applications in micro-electro-mechanical systems (MEMS) and nano-intelligent devices. For nano-structured materials, a growing body of research shows that several important physical properties, such as the elastic modulus (Chen et al., 2006; Jing et al., 2006), yield strength (Zhang et al., 2010), indentation hardness (Ma and Clarke, 1995; Feng and Nix, 2004) and melting temperature (Sun et al., 2002), become size-dependent; thus, determining how to interpret these interesting phenomena is being a hot point in solid mechanics and material science. At the nanoscale, the influence of the surface energy is of great importance because the surface to volume ratio is remarkably large for nanostructures, and quite a number of the size-dependent physical properties of nanosized materials can be rationalized by invoking the concept of surface energy.

Many researchers have studied the mechanical behaviors of the nano-structured materials by employing the surface stress theory (Gurtin and Murdoch, 1975, 1978; Povstenko, 1993; Cammarata, 1994; Huang and Wang, 2006; Huang and Sun, 2007). Miller and Shenoy (2000) studied the size-dependent elastic properties of nanosized structural elements and constructed a simple model to predict the size dependence of the effective properties. Sharma and Ganti (2004) and Duan et al. (2005a) studied the eigenstrain problem of spherical inhomogeneities with the interface effect and concluded that the Eshelby tensor is size-dependent. Dingreville et al. (2005) constructed a framework to incorporate the surface free energy and derived the effective moduli of the nanosized structural elements. Duan et al. (2005b) studied the effective elastic constants of composites that contained spherical nano-inhomogeneities with interface stress but they only considered the effect of the interface elasticity. Later, Huang and Sun (2007) established a micromechanical scheme to predict the effective modulus of nanocomposites, in which both the effect of the residual interface stress and the interface elasticity can be taken into account. It was shown that Duan et al. (2005b)'s result is just a special case of Huang and Sun (2007). Park and Klein (2008) investigated the surface stress effect on the resonant properties of nanowires and emphasized the importance of the residual surface stress. Dingreville and Qu (2008) derived a new relation between the interfacial excess energy and the interfacial excess stress for planar interfaces, which can account for both the in-plane and transverse deformations of the real material interfaces. Recently, several new directions in the surface effect have been explored. For example, the mechanics of rough surfaces and its applications were studied (Weissmuller and Duan, 2008; Mohammadi et al., 2013); the curvature dependence of the surface energy was considered to investigate its significance on nanostrucutres (Chhapadia et al., 2011;

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Mohammadi and Sharma, 2012); the surface effects were found to strongly influence the electromechanical coupling behaviors of nano-materials (Dai et al., 2011; Dai and Park, 2013).

There has been some preliminary research in contact mechanics at the nanoscale. Wang and Feng (2007) studied the two dimensional half-space problems with the effect of the residual surface stress. Long et al. (2012) studied the effect of the residual surface stress on the two dimensional Hertzian contact problem, and later Long and Wang (2013) generalized their work to the three dimensional case. Zhao and Rajapakse (2009) studied the influence of the surface elasticity on the surface-loaded isotropic elastic layers. It has been demonstrated that the residual surface stress and the surface elasticity are two equally important aspects in the surface effect, but only one of these effects is considered in the above-mentioned works. Gao et al. (2013) established a non-classical formulation of the Boussinesq problem, in which both the residual surface stress and the surface elasticity were considered, and constructed a three dimensional Hertizian contact model with the surface effect. However, the contact models reviewed above are only concerned with the Hertzian contact model. In fact, the Van der Waals interaction between the ideal surfaces of two solids will result in the adhesion between elastic bodies. Thus, the adhesion effect is truly a surface/interface phenomenon. When it comes to the elastic contact problems at the nano- or microscale, the adhesion should be an indispensable factor (Zhao et al., 2003).

The pioneering work in the adhesive contact can be traced back to Bradley (1932), who first solved the adhesive contact between a rigid sphere and a rigid plane and gave the formula of the pull-off force. The theory of the adhesive contact between two elastic bodies was first established by Johnson et al. (1971) in their eponymous Johnson–Kendall–Roberts (JKR) theory, which is based on the balance between the elastic energies and the work of adhesion. The JKR theory predicted a compressive stress field near the central region of contact and a singular tensile stress field near the contact edges. On the other hand, Derjaguin et al. (1975) developed an alternative adhesive contact theory (Derjaguin–Muller–Toporov theory or DMT theory), in which the stress field keeps in the Hertz profile within the contact region while the intermolecular adhesion outside the contact region is considered. Later, it was pointed out by Tabor (1977) that the JKR model is more suitable for the contact between relatively large and soft bodies while the DMT theory is more suitable for the contact between small and rigid bodies. Maugis (1992) developed a more general theory describing the transition between the JKR and DMT theories by using the Dugdale model. There has been extensive research that is based on these profound and significant adhesive contact theories. For example, a generalized adhesive contact model that considered the influence of shot-range and long-range attractive forces both inside and outside the actual contact area was developed (Schwarz, 2003); the classical JKR theory was extended to anisotropic materials and a model of reversible adhesion was developed (Chen and Gao, 2007); the adhesive behavior of the power-law graded materials was studied (Chen et al., 2009a,b); the adhesion of the nanoscale asperities with power-law profiles was investigated (Zheng and Yu, 2007; Grierson et al., 2013). It should be noted that there are substantial significant results on the adhesive contact in the literature, but regretfully, we can only review a small part of them here. The reader may refer to Barthel (2008) for a review of the adhesive interactions in contact mechanics. At the small length scales, both the adhesion and the surface stress play important roles in MEMS and nano-intelligent devices. However, to the authors' knowledge, the effect of surface stress on the adhesive contact between elastic bodies at the nanoscale has not been studied.

The objective of the present paper is to generalize the classical JKR adhesive contact model to the nanoscale by considering the surface effect and investigate the influence of the surface stress on the adhesive contact. The non-classical Boussinesq fundamental solutions developed by the authors in a previous paper (Gao et al., 2013) are employed to formulate this non-classical adhesive contact model. It is found that, compared with the classical theory, the pull-off force increases while the corresponding critical contact radius decreases as a result of the surface effect. A detailed theoretical study of these significant phenomena is presented and two scaling laws are constructed based on dimensional analysis. These new scaling laws describe the characteristics of the adhesive contact at the nanoscale. It should be mentioned that, for simplicity, the surface roughness is not considered in the present work.

This paper is organized as follows. The basic theoretical framework of the JKR adhesive contact model with the surface effect is formulated in Section 2. The numerical results of the developed theory are illustrated in Section 3. The scaling laws of the pull-off force and the relevant critical contact radius are constructed in Section 4 using the dimensional analysis. The conclusions are summarized in Section 5.

2. Basic theory

The goal of this section is to generalize the classical JKR adhesive contact theory to the nanoscale by considering the surface effect. The non-classical Boussinesq solutions are given first as the preliminary, and then the basic theoretical framework of the JKR theory with the surface effect are formulated.

2.1. Non-classical Boussinesq solutions

The fundamental solutions of the Boussinesq problem play an important role in contact mechanics. At the nanoscale, a non-classical formulation of the Boussinesq problem with the surface stress effect was developed by Gao et al. (2013). In the three-dimentional Boussinesq problem, the normal displacement solution with the surface effect under axisymmetric normal pressure $p(r)$ is

$$u_z = \frac{1}{2\mu}\int_0^\infty \frac{\overline{p}(\xi)}{g(\xi)}[2(4\nu-3)k\xi + 8(\nu-1) - 2(k\xi+2)\xi z]e^{-z\xi}J_0(\xi r)\mathrm{d}\xi, \tag{1}$$

where μ and ν are the shear modulus and the Poisson ratio of the material, respectively, $J_0(\xi r)$ denotes the zero order Bessel function of the first kind and

$$\overline{p}(\xi) = \int_0^\infty rp(r)J_0(\xi r)\mathrm{d}r \tag{2}$$

is the Hankel transformation of the normal pressure $p(r)$. The function $g(\xi)$ is expressed as

$$g(\xi) = -4 + 4(\nu-1)(k+l)\xi + (4\nu-3)kl\xi^2 \tag{3}$$

and l and k are two intrinsic length scales, which reflect the surface effect and are defined as

$$l = \frac{\sigma_0}{\mu}, \quad k = \frac{\gamma_1^* + \gamma_1}{\mu}, \tag{4}$$

where σ_0 denotes the residual surface stress and γ_1^*, γ_1 are elastic constants of the material surface. For details, the reader may refer to the references by Huang's group (Huang and Wang, 2006, 2013; Huang and Sun, 2007).

Putting $z = 0$ in the Eq. (1), we obtain the surface displacement under normal pressure:

$$\overline{u}_z(r) = \frac{1}{2\mu}\int_0^\infty \overline{p}\left(\frac{t}{a}\right)\frac{2(4\nu-3)k_r t + 8(\nu-1)}{g(t;l_r,k_r)}J_0\left(\frac{r}{a}t\right)\mathrm{d}t, \tag{5}$$