International Journal of Solids and Structures 51 (2014) 575–598

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A higher order model for thin-walled structures with deformable cross-sections

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ARTICLE INFO

Article history:
Received 22 November 2012
Received in revised form 14 October 2013
Available online 26 October 2013

Keywords:
Thin-walled structures
Higher order deformation modes
Warping
Distortion

ABSTRACT

A higher order model for the analysis of linear, prismatic thin-walled structures that considers the cross-section warping together with the cross-section in-plane flexural deformation is presented in this paper. The use of a one-dimentional model for the analysis of thin-walled structures, which have an inherent complex three-dimensional (3D) behaviour, can only be successful and competitive when compared with shell finite element models if it fulfills a twofold objective: (i) an enrichment of the model in order to as accurately as possible reproduce its 3D elasticity equations and (ii) the definition of a consistent criterion for uncoupling the beam equations, allowing to identify structural deformation modes.

The displacement field is approximated through a linear combination of products between a set of linear independent functions defined over the cross-section and the associated weights only dependent on the beam axis; this approximation is not constrained by any *ab initio* kinematic assumptions. Towards an efficient application of the approximation procedure, the cross-section is discretized into thin-walled elements, being the displacement field approximated for each element independently of the displacement direction. The approximation is thus *hp* refined enhancing the "capture" of the 3D structural mechanics of thin-walled structures. The beam model governing equations are obtained through the integration over the cross-section of the corresponding elasticity equations weighted by the cross-section global approximation functions.

A criterion for uncoupling the beam governing equations is established, allowing to (i) retrieve the classic equations of the thin-walled beam theory both for open and closed sections and (ii) derive a set of uncoupled deformation modes representing higher order effects. The criterion is based on the solution of the polynomial eigenvalue problem associated with the beam differential equations, allowing to quantify the Saint-Venant principle for thin-walled structures. In fact, the solution of the non linear eigenvalue problem yields a twelve fold null eigenvalue (representing polynomial solutions) that are verified to represent beam classic solutions and sets of pairs and quadruplets of non-null eigenvalues corresponding to higher order modes of deformation.

1. Introduction

The analysis of prismatic thin-walled structures through one-dimensional models represents a simple and efficient procedure that has been successfully adopted. This trend of analysis has evolved through the development of refined beam models in order to represent the corresponding three-dimensional structural behaviour side by side with the deployment of an increasingly enhanced technology of shell finite elements available to model such structures.

However, the use of shell finite elements, although being inherently accurate in modelling the 3D structural behaviour of thin-walled structures, is not only costly in terms of computing resources, but also presents a cumbersome set of results that can difficult the interpretation of the relevant phenomena from less experienced users. Moreover, the modelling by shell finite elements requires a significant amount of data that remains unknown at a preliminary stage of design, being also more prone to modelling errors from users when compared with beam models.

The formulation of beam models by reducing the 3D elasticity formulation to a one-dimensional model must be as accurate as possible in order to include the most significative structural phenomena. Towards this end, several trends of enhancing beam theories are identified: asymptotical methods, in particular variational asymptotical beam sectional analysis (VABS) by Yu et al. (2012), expansion of the beam displacement field through Taylor series (Carrera and Giunta, 2010; Carrera et al., 2011; Carrera and Petrolo, 2011, Saint-Venant driven models (Toupin, 1965; Knowles, 1966; Cowper, 1966; Horgan, 1989; Giavotto et al., 1983; Bauchau,

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http://dx.doi.org/10.1016/j.ijsolstr.2013.10.023

1985; Laudiero and Savoia, 1990; Rubin, 2003; Ladevèze et al., 2003; Fatmi, 2007; Morandinia et al., 2010) and refinement of the classic thin-walled theory which was cast through the seminal works of Umansky (1940), von Kármán and Christensen (1944), Flügge and Marguerre (1948) and Vlassov (1961) towards the consideration of the cross-section in-plane deformation and additional degrees of warping (Bazant, 1968; Bazant and El Nimieri, 1974) and "additional" deformations modes in order to combine the warping due to torsion and related with shear-lag effects together with the cross-section deformation flexure (Mikkola and Paavola, 1980; Paavola, 1990; Tesar, 1996; Razaqpur and Li, 1991, 1994; Prokic, 1996a,b, 2002 and Kim and Kim, 1999a,b, 2000, 2002, 2003).

Some other formulations stemming from the classic thin-walled beam theory have been developed by considering the addition of "representative" deformation modes that were obtained through simplified models that are often thought for specific structural behaviours and/or load conditions. The analysis of the cross-section distortion in box girders through an analogy of a beam on an elastic foundation is a successful example of these models, Wright and Abdel-Samad (1968) and Hsu (1995) as well as the analysis of the distortion presented in Boswell and Zhang (1984) and Kermani and Waldron (1993) and more recently solutions for multi-cell distortion given by Pavazza (2002), Pavazza and Blagojević, 2005 and Park et al. (2005); and the warping due to the shear-lag effects (Kuzmanovic and Graham, 1981; Foutch and Chang, 1982 and Dezi and Mentrasti, 1985). The in-plane distortion of thin-walled structures has been also was considered through the definition of distortional modes (Saadé et al., 2006).

A beam theory that considers both the warping and the cross-section distortion, allowing to derive a set of the corresponding modes is the generalised beam theory (GBT) that since its inception in Schardt (1966) and Schardt (1989) has been subjected to several developments. In fact, Initially, the theory did not allow to consider multi-cell closed cross-sections and open cross-sections with more than two walls converging in a node, which has been coped through new formulations by Möller (1982), Simão (2005) and Gonçalves (2007) and more recently to arbitrary sections in Dinis et al. (2006), Gonçalves et al. (2009) and Gonçalves et al. (2010). The theory has also been developed to account for shear deformation and transverse extension, being also applied to composite materials in Silvestre and Camotim (2002a) and Silvestre and Camotim (2002b).

A definition of the distortional displacement field within the framework of GBT has been presented and applied in Jönsson and Andreassen (2011, 2012) and Andreassen and Jönsson (2013). This novel approach considers a cross-section discretization into elements, which includes rotational degrees of freedom for the transverse displacements perpendicular to the wall, and from an approximation similar to the GBT formulations establishes a governing equation for the beam model. The displacement field displacement approximation considers the amplitude of the warping displacement shape functions to be the derivative of the transverse displacement amplitudes. This fact together with shear constraints allowed to bind together axial and transverse displacements, which was adopted for the definition of warping functions from the transverse displacements and for rewriting the beam governing equations in terms of transverse displacements.

The formulation considers a strategy of eliminating classic solutions (axial pure modes, translational modes and rotational modes) from the beam governing equations in order to derive a set of equations that defines distortional behaviour. The pure axial extension is identified as a solution of the equations and a linear solution is put forward since the amplitude for the axial mode is admitted to correspond to the derivative of the transverse displacements. Despite referring to the pure axial mode as an eigenmode solution, no statement of the corresponding eigenvalue is presented and hence the corresponding polynomial solution considered does not identify the respective generalised eigenvectors. By eliminating the axial mode, the set of beam equations is written in terms of transverse displacements, which is adopted to identify the transverse rigid cross-section translations and the pure rotational mode. The translational modes are obtained from the eigenvectors of a standard eigenvalue problem applied to the axial stiffness matrix, being the rotational mode defined by requiring that the coupling terms in the axial stiffness cancel. Furthermore, the membrane tangential displacements along each wall element are set to be equal so as to enforce the transverse strain of the middle surface null. The beam equations are rewritten for a new set of coordinates that consider the translational modes, the rotational modes and the tangential constraint, being tested a polynomial cubic solution for the translational modes.

The procedure adopted for identifying and in the sequel eliminate the so-called axial, translational and rotational modes is quite different from that applied in Vieira et al. (2013). In fact, the classic solutions of beam theory, including shear effects, are obtained through the eigenvectors and generalised eigenvectors associated with a 12-fold null eigenvalue, which is a solution of the non-linear eigenvalue associated with the beam model differential equations. Moreover, the correct procedure for eliminating these classic modes from higher order modes is essential to obtain an isospectral transformation of modes.

On the other hand, the GBT formulation of Jönsson and Andreassen (2011) considers a different strategy for identifying the classic modes. In fact, the null eigenvalue is identified as the solution associated with classic modes, but the polynomial solutions put forward for each of the classic modes are not demonstrated to correspond to the respective generalized eigenvectors: As a result the solution lacks of consistency insofar as the corresponding algebraic and geometric multiplicities do not coincide. In fact, in the examples presented in Jönsson and Andreassen (2011) the classic modes are associated with a null eigenvalue with an algebraic multiplicity of 8, despite having a 12-fold geometric multiplicity. In the proposed higher order model the classic solution stems directly from the computation of the Jordan chain associated with a 12-fold null eigenvalue, clearly identifying extension, flexure (with and without shear) and torsion classic structural behaviour, but also rigid body motions.

A semi-analytical finite strip analysis (the constrained finite strip analysis – cFSM) has been able to shed a light regarding the mechanical behaviour of thin-walled structures through the separation of the corresponding deformation modes (Li et al., 2011; Ádány and Schafer, 2008). A comparison between the modal approaches between GBT and is cFSM has been presented in Ádány et al. (2009).

A one-dimensional model for the three-dimensional structural analysis of prismatic thin-walled structures with an arbitrary midline geometry considering the cross-section warping together with its transverse deformation is presented in this paper. This higher order model represents a novel beam formulation capable of representing the three-dimensional structural behaviour of thin-walled structures, being an alternative to other theories such as those of Pavazza and Blagojević (2005), Dinis et al. (2006), Saadé et al. (2006), Gonçalves et al. (2010) and Carrera and Giunta (2010). The beam model is thought to be cast within the framework of the finite element method and derived in a unified formulation (i.e. the formulation is independent of the cross-section type).

A previous papers by the authors (Vieira et al., 2013) considers the cross-section in-plane undeformable in order to obtain a set of warping and shear modes. A more general formulation including in-plane deformation is now proposed with a particular focus on the definition of uncoupled modes. The uncoupling procedure is derived from a quartic eigenvalue problem, which given its