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Rolling resistance of a rigid sphere with viscoelastic coatings

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ABSTRACT

We present a novel three-dimensional boundary-element formulation that fully characterizes the mechanical behavior of the external boundary of a multi-layered viscoelastic coating attached to a hard rotating spherical core. The proposed formulation incorporates both, the viscoelastic, and the inertial effects of the steady-state rolling motion of the sphere, including the Coriolis effect. The proposed formulation is based on Fourier-domain expressions of all mechanical governing equations. It relates two-dimensional Fourier series expansions of surface displacements and stresses, which results in the formation of a compliance matrix for the outer boundary of the deformable coating, discretized into nodes. The computational cost of building such a compliance matrix is optimized, based on configurational similarities and symmetry. The proposed formulation is applied, in combination with a rolling contact solving strategy, to evaluate the viscoelastic rolling friction of a coated sphere on a rigid plane. Steady-state results generated by the proposed model are verified by comparison to those obtained from running dynamic simulations on a three-dimensional finite element model, beyond the transient. A detailed application example includes a verification of convergence and illustrates the dependence of rolling resistance on the applied load, the thickness of the coating, and the rolling velocity.

1. Introduction and background

Viscoelasticity is a time-dependent model of material behavior capable of replicating the storage and restitution (elasticity), and the dissipation in the bulk at different internal rates (viscosity), of variable proportions of the deformation energy. Particles and solids of rounded shape presenting viscoelastic properties, or interacting mechanically with other viscoelastic entities, with or without direct contact, are involved in many aspects of sciences and technologies, in various fields, and at different length-scales, from the smallest fundamental particles (e.g. Berg, 1999), to nano-materials and living cells (e.g. Bahadur and Schwartz, 2008; Bose et al., 2010; Coghill, 2012; Subramaniam et al., 2013; Xu and Shao, 2008), to various sorts of objects and systems at the human scale, such as the motion of rigid spheres in polymer gels (e.g. Hunter, 1968), the vibratory sorting of fruits and vegetables (e.g. Arnold, 1985), polymer-coated grinding spheres (e.g. Langus et al., 2011), rubber bullets (e.g. Bir et al., 2012), particle dampers (e.g. Els, 2009), structural damping fillers (e.g. Oyadiji, 1996), computer mouse-balls, spherical wheels for vehicles and robots (e.g. Wu and Hwang, 2008; Wu et al., 2011), flows of viscoelastic fluids around spheres (e.g. Atsbha, 1993), flows of granular materials (e.g. Yung et al., 2007; Zhou et al., 1999), human or animal joints (e.g. Esat and Ozada, 2010), and rolling balls in seismic isolation platforms (e.g. Harvey et al., 2013; Tsai et al., 2010), to the largest planets, and stars (e.g. Bambusi and Haus, 2012).

Among all possible types of static or dynamic interactions between one, two, a few, or even very large numbers of rounded entities, those involving contact are very common, and often accompanied by losses of mechanical energy. Upon rolling or sliding, mechanical energy is transformed into heat in the continuum of those of the interacting objects that are characterized by a viscoelastic behavior. This dissipative process in the bulk, known as viscoelastic "rolling resistance", or viscoelastic "rolling friction", is reflected by changes in the mechanical fields (i.e. the stresses and strains) across the contact interfaces, so as to resist the ongoing motion.

Problems related to the resistance incurred by rigid indenters, such as cylinders, spheres and cones, rolling or sliding on a viscoelastic plane, are addressed quite extensively in the scientific literature, both experimentally and from a modeling perspective, in two and three dimensions, and at different scales, such as in the works of Bueche and Flom (1959), Chertok and Putignano (2013), Chertok et al. (2001), Flom and Bueche (1959), Flom (1960), Galin and Gladwell (2008), Greenwood and Tabor (1958), Greenwood et al. (1961), Hunter (1961), Johnson (1985), Lee et al. (2009), May et al. (1959), Persson (2010), Pöschel et al. (1999), Qiu (2006), Tabor (1952, 1955) and Zéhil and Gavin (2013a,b,c), to cite a few. Alternatively, the rolling contact between viscoelastic cylinders, or between a viscoelastic cylinder and a rigid plane is analyzed for instance by Golden and Graham (2001), Kumar et al.

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(1988), Morland (1967, 1968), Munisamy et al. (1991), Nowell and Hills (1988), Oden and Lin (1986), Qiu (2009) and Wang and Knothe (1993), while Hall (2001) presents a nice review of the fundamentals of rolling resistance from the perspective of the tire industry. However, quite surprisingly, problems involving the rolling/sliding friction of viscoelastic spheres have received far less attention. In the following, we briefly review four studies that focus on the rolling resistance incurred by a *solid* viscoelastic sphere on a rigid plane, mostly, in narrow ranges of the parameters.

Brilliantov and Pöschel (1998) propose an approximate closed-form expression for the friction coefficient of a solid viscoelastic sphere of radius R rolling on a hard plane, in quasi-static conditions and under small deformations. The authors assume that the characteristic time of the motion, defined as the ratio of the sphere's deformation $d = R - H$ (H being the distance between the center of the deformed sphere and the contact surface) to its rolling velocity V_s, is much larger than the material's internal time scales τ_k. In agreement with this regime of motion and with the small strain assumption $d/R \ll 1$, inertial forces are neglected and the vertical displacement field is approximated by the corresponding result of the stationary contact problem, as given by Hertz (1881). The behavioral characterization of the viscoelastic material is limited to two viscous parameters, or equivalently, to a single relaxation time for each of the shear modulus and the bulk modulus. In fact, the stress field is written as the sum of an elastic part $\boldsymbol{\sigma}_e$, and a viscous part $\boldsymbol{\sigma}_v$, which corresponds to the Kelvin–Voigt model, characterized by a constant storage modulus, and by a loss modulus increasing linearly with frequency. This choice is consistent with the other assumptions retained by the authors in the sense that the linearization of any frequency-domain viscoelastic master-curves about zero frequency, corresponds to a Kelvin–Voigt model. This fact is readily inferred, for instance, from their Eqs. (65a) and (65b). It is assumed that the elastic part of the contact stress field is almost unaffected by the (slow) motion, and that it remains roughly symmetrical. Its contribution to the resisting torque T_r is hence neglected, in comparison to that of the viscous stress field. The authors show that, within the framework of the proposed theory, the resisting torque scales linearly with the vertical load applied to the rolling sphere P, with its radius R, its angular speed Ω, and therefore with its velocity $V_s \approx R\Omega$. The proposed expression is however flawed due to an error in a coordinate system transformation, as recently determined by Zheng et al. (2011).

A few years later, Yung and Xu (2003) argue that, in most practical cases, the material's internal rates of dissipation cannot be considered much smaller than the characteristic time of motion, and therefore conclude that more accurate expressions are needed for the rolling resistance of viscoelastic spheres, which take into account the "influence of relaxation". To this aim, the authors 'relax' the assumption $d/V_s \gg \tau_k$, attempting to reveal the nonlinear dependence of rolling resistance on velocity, at moderately higher rates of motion. They however stipulate, for simplicity, that the fields in the continuum of the sphere, at a given cycle, are not influenced by the preceding cycles, which is equivalent to maintaining the limiting condition that $\Omega\tau_k \ll 1$. It is interesting to note that the latter constraint is satisfied implicitly under the assumptions retained earlier by Brilliantov and Pöschel (1998), i.e. $\Omega\tau_k \ll d/R \ll 1$. In contrast, Yung and Xu (2003)'s assumptions that $\Omega\tau_k \ll 1$ and that $d/R \ll 1$ are unconnected, which expands the applicability domain of their theory to the nonlinear regime, by increasing the upper bound on Ω. In deriving a nonlinear relation for rolling resistance, the authors make several other simplifying assumptions, some of which are quite limiting, and somewhat inconsistent with their stated goal, such as retaining one Kelvin–Voigt element to model the material's behavior. Indeed, this material model is characterized by a single rate of internal dissipation and is known to better reflect creep than relaxation. Other approximations include: (i) introducing the viscous behavior vertically and pointwise (ii) neglecting inertial effects under the quasi-static approximation, (iii) retaining the same contact radius r_c and deformation d as the stationary Hertzian solution, (iv) assuming a sinusoidal stress distribution across the contact surface, calibrated to yield the same maximum contact pressure as that of the stationary solution, (v) evaluating an 'average' density of dissipated energy at one point of 'average' position with 'average' values of the fields, and (vi) evaluating the total dissipation by integration over a "deformed volume" $2\pi R r_c d$ of ring-like cross section defined by the contact path $2\pi R$, the 'average' contact width r_c, and the deformation d. The resulting analytical expression for rolling resistance is quite cumbersome. A numerical example reveals that, according to the proposed theory, rolling friction first increases, then decreases, with increasing velocity. However, given the constitutive model retained, and in the absence of inertial effects, the physical mechanisms causing the rolling friction to decrease with increasing speed is rather unclear.

Xu et al. (2007) present an experimental apparatus that measures the steady-state coefficient of rolling friction $T_r/(PR)$ of a squash ball on a conveyor belt, at moderate velocities. The setup was designed to fill an identified gap in the availability of accessible methods to perform rolling resistance experiments involving deformable spheres. It was later used in a classroom for teaching purposes. The different sources of power dissipation contributing to rolling resistance cannot be clearly distinguished using the proposed device. Indeed, energy losses occur not only in the bulk of the sphere, but also to some extent in the bulk of the deformable conveyor belt, and at the contact interface in case of slipping friction as well. Nevertheless, the experimental results presented by the authors, for the combined losses, confirm the linear dependence of the coefficient of rolling friction on the translational velocity V_s, at moderate rates of motion.

More recently, Zheng et al. (2011) implement using the commercial software ABAQUS, a finite element (FE) model for the steady-state rolling resistance of a solid viscoelastic sphere on a rigid plane, under the quasi-static approximation. The material's behavior is characterized as in the work of Brilliantov and Pöschel (1998), with the additional assumption that the viscous parameter associated with the bulk modulus is equal to zero, which in fact corresponds to the three-dimensional formulation of the material behavior retained by Yung and Xu (2003). The FE model's implementation is focused on the regime where $\Omega\tau_k \ll 1$, which corresponds to the elastic part of the contact stress field being much larger than the viscous part, but also to small values of rolling resistance. To avoid that the resisting toque be affected by numerical errors on $\boldsymbol{\sigma}_e$, the authors override the finite element software to compute the resisting torque from $\boldsymbol{\sigma}_v$ only, hence neglecting the contribution of $\boldsymbol{\sigma}_e$. The numerical model is exploited in the conditions corresponding to $\Omega\tau_k \ll d/R$ and to $d/R \ll 1$. In this regime, the coefficient of rolling friction, defined as $T_r/(PR)$, is found to be almost independent from the vertical load P applied to the sphere, and to vary linearly with the rolling speed Ω. The authors also derive an analytical expression for rolling friction, based on the assumptions retained by Brilliantov and Pöschel (1998), which matches the results of their numerical model fairly well.

It is interesting to note that, in the papers discussed above, either (i) a simplified formulation is retained which does not involve a rolling contact problem, or (ii) a more sophisticated numerical approach is adopted, but the resolution of contact is carried out by means of a commercially available tool, with minimal discussion.

To date, no work has ever addressed the modeling (and the solving) of the resistance incurred by a rigid sphere, covered with a viscoelastic coating, rolling or sliding, on a rigid plane. In this work, we present a novel three-dimensional boundary-element