International Journal of Solids and Structures 51 (2014) 848–860

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

The role of electrostriction on the stability of dielectric elastomer actuators

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ARTICLE INFO

Article history:
Received 23 March 2013
Received in revised form 24 September 2013
Available online 27 November 2013

Keywords:
Electroelasticity
Electroactive polymers
Smart materials
Electromechanical instability
Buckling actuator

ABSTRACT

In the field of soft dielectric elastomers, the notion ‘electrostriction’ indicates the dependency of the permittivity on strain. The present paper is aimed at investigating the effects of electrostriction onto the stability behaviour of homogeneous electrically activated dielectric elastomer actuators. In particular, three objectives are pursued and achieved: (i) the description of the phenomenon within the general nonlinear theory of electroelasticity; (ii) the application of the recently proposed theory of bifurcation for electroelastic bodies in order to determine its role on the onset of electromechanical and diffuse-mode instabilities in prestressed or prestretched dielectric layers; (iii) the analysis of band-localization instability in homogeneous dielectric elastomers. Results for a typical soft acrylic elastomer show that electrostriction is responsible for an enhancement towards diffuse-mode instability, while it represents a crucial property – necessarily to be taken into account – in order to provide a solution to the problem of electromechanical band-localization, that can be interpreted as a possible reason of electric breakdown. A comparison between the buckling stresses of a mechanical compressed slab and the electrically activated counterpart concludes the paper.

1. Introduction

Dielectric elastomer (DE) devices are electrically activated smart systems that possess mechanical properties similar to those of natural muscles and therefore represent one of the most promising members within the class of the artificial muscles (Bar-Cohen, 2001; Brochu and Pei, 2010). Applications of these systems are common in the fields of mechatronics, aerospace, biomedical and energy engineering as actuators, sensors, and energy harvesters (Carpi et al., 2008a). Their operating principle is based on the deformation of a dielectric soft membrane induced by the electrostatic attraction forces arising between the charges placed on its opposite sides (Pelrine et al., 1998, 2000); such effect is proportional to the permittivity of the material, which unfortunately turns out to be very low for the typical materials in use (e.g., silicones, acrylic elastomers) with relative dielectric constants ϵ_r amounting to a few units.

While, on the one hand, research efforts are devoted to the design and realization of composite materials with significantly higher permittivities to improve the electromechanical coupling (Zhang et al., 2002; Huang et al., 2004; deBotton et al., 2007; Carpi et al., 2008b; Molberg et al., 2010; Bertoldi and Gei, 2011; Risse et al., 2012; Ponte Castaneda and Siboni, 2012; Tian et al., 2012; Gei et al., 2013), on the other hand, the nonlinear theory of homogeneous soft dielectrics is still under way, in particular, special attention deserve the issues associated with the different types of instability developing under operating conditions and those related to the intrinsic behaviour of the material, such as electrostriction and polarization saturation (Li et al., 2011a; Ask et al., 2012, 2013).

The aim of this paper is to give a contribution to the aspects just mentioned, by pursuing three main goals:

- to provide a framework accounting for electrostriction of soft DEs within the general nonlinear theory of electroelasticity. As usual in the field of soft dielectrics, electrostriction is conceived as the dependency of the relative permittivity of the material on strain: this effect, experimentally observed (Wissler and Mazza, 2007; Li et al., 2011b), must be taken into account for modelling purposes in view of the large deformations usually achieved. This phenomenon has been theoretically addressed by Zhao and Suo (2008) who employed a simple model for its characterization;
- to apply the general theory of bifurcation for electroelastic body proposed by Bertoldi and Gei (2011) to investigate (i) electromechanical instability in unconstrained specimens and (ii) diffuse-mode instabilities, including buckling-like and surface-like modes, in prestretched dielectric layers. In the aforementioned

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0020-7683/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijsolstr.2013.11.011

paper the focus was on layered composites (see also Nobili and Lanzoni, 2010, and Rudykh and deBotton, 2011), while the current analyses are performed on homogeneous materials, for which the two types of bifurcation are obtained on the basis of a common general criterion. Electromechanical instability on its own was extensively studied by methods developed by Zhao et al. (2007) and De Tommasi et al. (2010), while De Tommasi et al. (2013) showed that an imperfection could trigger this instability at a voltage much lower than that for a homogeneous specimen. Regarding the importance of diffuse modes, we mention that an Euler-like instability is the activation mechanism of several types of buckling-like actuators (Carpi et al., 2008a; Vertechy et al., 2012);

- to analyze band-localization instability in homogeneous DEs, in particular facing its relation with the constitutive properties of the solid. The theory developed here extends to the electroelastic domain the well-known theory of localization of deformation in nonlinear elasticity, where the existence of a localized solution of the incremental problem – concentrated within a narrow band – is sought along the loading path (Rice, 1973; Hill and Hutchinson, 1975; Bigoni and Dal Corso, 2008).

General assumptions adopted throughout the paper involve plane-strain deformation and material incompressibility. Fig. 1 reports a sketch of the investigated instabilities relevant to the homogeneous loading paths assumed for the layer that is always actuated by a given transverse electric displacement field: in the first (Path A), the prestressed specimen can freely expand under the electrical actuation, while in the second (Path B), the layer is first mechanically prestretched and successively actuated. The obtained results, well suited to a wide class of diffused acrylic elastomers, show that electrostriction plays a fundamental role in the stability behaviour of the actuators.

The paper is organized into eight sections. Sections 2 and 3 deal with the formulation of the finite and the linearized electroelastic models, respectively. Section 4 introduces the considered electromechanical loading paths, while in Section 5 the formulation of the general theory of electroelastic bifurcations introduced by Bertoldi and Gei (2011) is recalled and specialized to the plane-strain problem under study. The band-localization instability is discussed in Section 6, while all results and their interpretation are presented in Section 7. Finally, the conclusions are summarised in Section 8, while in Appendix A the components of the incremental moduli associated with the general free energy introduced in Section 2 are detailed.

2. Large deformations and stress state for a soft dielectric body

Here the theory of large-strain electroelasticity for a homogeneous isotropic hyperelastic body is briefly recalled, on the basis of the notion of total stress. The reader is referred to McMeeking and Landis (2005), Dorfmann and Ogden (2005), Suo et al. (2008) and Bertoldi and Gei (2011) for further details.

A system in equilibrium under external electromechanical actions is considered, including an electroelastic body occupying a region $B \in \mathbb{R}^3$, whose points are denoted by $\boldsymbol{x}$ and the surrounding space $B^{sur} = \mathbb{R}^3 \setminus B$. Here the general case of the surrounding domain occupied by a different dielectric medium is briefly illustrated, while in our reference problem we assume B^{sur} corresponding to vacuum, in such case it will be denoted by B^*. The stress-free configuration of the body B^0, whose points are labelled $\boldsymbol{x}^0$, can be identified, such that $\boldsymbol{x} = \boldsymbol{\chi}(\boldsymbol{x}^0)$, where $\boldsymbol{\chi}$ represents a given deformation and $\boldsymbol{F} = \partial\boldsymbol{\chi}/\partial\boldsymbol{x}^0$ denotes its gradient. In the general case, the material configuration of the surrounding domain is analogously denoted by $B^{0^{sur}}$, while no reference configuration is introduced in the case of vacuum, as the deformation gradient is not defined there.

2.1. Field equations and boundary conditions

Under the hypotheses previously introduced and assuming the absence of body forces and volume free charges, the governing equations of the system in the spatial description are:

$$\operatorname{div}\boldsymbol{\tau} = \boldsymbol{0}, \quad \boldsymbol{\tau}^T = \boldsymbol{\tau}, \quad \operatorname{div}\boldsymbol{D} = 0, \quad \operatorname{curl}\boldsymbol{E} = \boldsymbol{0} \quad (\text{in } B \cup B^{sur}). \tag{1}$$

Here $\boldsymbol{\tau}$ denotes the 'total' stress, while $\boldsymbol{D}$ and $\boldsymbol{E}$ represent the electric displacement and the electric field respectively; operators written with initial lower-case (upper-case) refer to variables defined in the present (reference) configuration. Condition (1)$_4$ states that $\boldsymbol{E}$

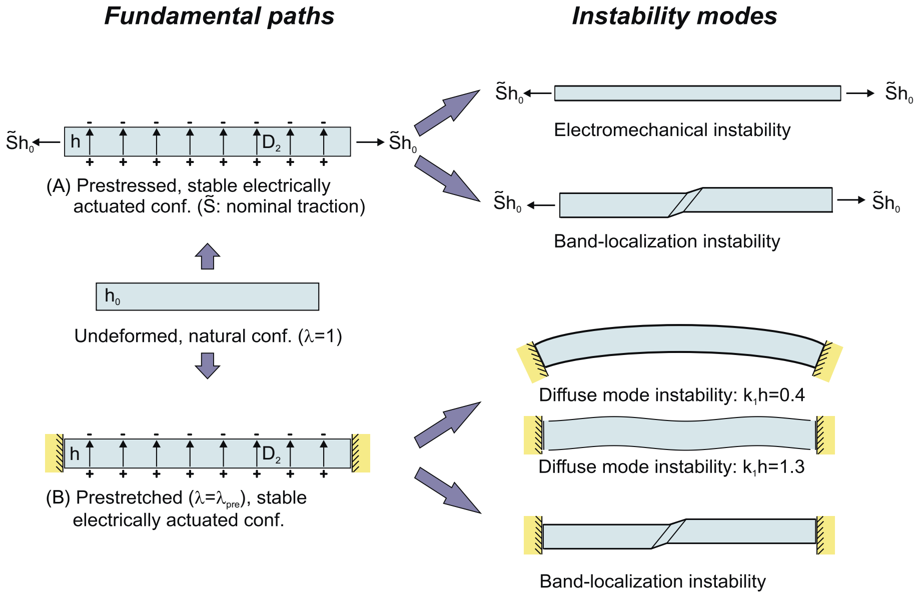

Fig. 1. Sketch of the instabilities investigated in the paper for a soft dielectric layer subjected to two different electromechanical loading paths. A: the layer is electrically actuated in the presence of a constant longitudinal force –$\tilde{S}$ is the nominal traction–; B: the layer is first prestretched at a longitudinal stretch equal to λ_{pre} and then electrically actuated (h_0 and h denote the initial and the reference thicknesses, respectively; D_2 represents the current electric displacement field).