International Journal of Solids and Structures 51 (2014) 861–874

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

New analytical criterion for porous solids with Tresca matrix under axisymmetric loadings

Oana Cazacu [a,*], Benoit Revil-Baudard [a], Nitin Chandola [a], Djimedo Kondo [b]

[a] Department of Mechanical and Aerospace Engineering, University of Florida, REEF, 1350 N. Poquito Rd, Shalimar, FL 32579, USA
[b] Universite Pierre et Marie Curie, Institut D'Alembert, UMR 7190, CNRS, 4, Place Jussieu, 75252 Paris, France

ARTICLE INFO

Article history:
Received 28 January 2013
Received in revised form 4 November 2013
Available online 21 November 2013

Keywords:
Ductile porous solids
Coupled third-invariant mean stress effects
Limit analysis
Tresca yield criterion

ABSTRACT

In this paper, a new analytic criterion for porous solids with matrix obeying Tresca yield criterion is derived. The criterion is micromechanically motivated and relies on rigorous upscaling theorems. Analysis is conducted for both tensile and compressive axisymmetric loading scenarios and spherical void geometry. Finite element cell calculations are also performed for various triaxialities. Both the new model and the numerical calculations reveal a very specific coupling between the mean stress and the third invariant of the stress deviator that results in the yield surface being centro-symmetric and void growth being dependent on the third-invariant of the stress deviator. Furthermore, it is verified that the classical Gurson's criterion is an upper bound of the new criterion with Tresca matrix.

1. Introduction

Significant progress has been made in understanding and modeling the micromechanics of ductile fracture in porous polycrystalline metallic materials. Most of the available theories of dilatational plasticity and viscoplasticity make use of the assumption that the matrix (void-free material) is described by the von Mises yield criterion. This is the case, for example, of the classical Gurson (1977) model and its various extensions proposed by Tvergaard (1981), Tvergaard and Needleman (1984), Gologanu et al. (1993), Monchiet et al. (2011) among others. In particular, the modified Gurson model, known as Gurson–Tvergaard–Needleman (GTN) model reproduces qualitatively the essential features of tensile fracture of axisymmetric specimens (e.g. Tvergaard and Needleman, 1984; Koplik and Needleman, 1988). All the above models involve dependence only with the mean stress and the von Mises effective stress. However, theoretical studies have revealed that triaxiality alone is insufficient to characterize important growth and coalescence features even for axisymmetric stress states (e.g. Ponte Castañeda and Suquet, 1998 for weakly contrasted materials, Danas et al., 2008, who applied the second-order method of Ponte Castaneda, 2002 to the case of a porous von Mises material). Further evidence of combined effects of the mean stress and third invariant on yielding of porous solids with von Mises matrix were also provided using finite-element (FE) cell calculations (e.g. Cazacu and Stewart, 2009; Julien et al., 2011; Thore et al., 2011, etc.). Very recently, Cazacu et al. (2013) developed an analytic yield criterion that captures the aforementioned trends under axisymmetric stress states, namely the centro-symmetry of the yield surface and the role of the sign of the third-invariant on the rate of void growth (see Alves et al., 2013).

As concerns the ductile response of porous metals under shear dominated loadings (at low triaxialities), in the past couple of years, growing experimental evidence has shown the role played by all stress invariants. In particular, the influence of the Lode parameter has been well documented (e.g. recent data reported by Bao and Wierzbicki (2004), Barsoun and Faleskog (2007), Haltom et al. (2013) and Lou and Huh (2013), etc.). This dependence has also been investigated from theoretical /computational standpoints (e.g. Nashon and Hutchinson, 2008; Tvergaard, 2009; Tvergaard and Nielsen, 2010; Stoughton and Yoon, 2011, etc.).

Since in all the above studies, the fully-dense material is described by the von Mises yield criterion, the effects of the third-invariant of the stress deviator on the dilatational response of the porous solid are due solely to the presence of voids. Cazacu and Stewart (2009) developed an analytical potential for porous solids for which the matrix is incompressible but displays tension–compression asymmetry (e.g. hcp porous solids). Specifically, the yield criterion used for the matrix is an odd function of the stress deviator and involves dependence on its two invariants. It was shown that the yield surface of the porous solid does not display any symmetries with respect to the deviatoric and hydrostatic axes, respectively (see also Lebensohn and Cazacu, 2012).

In this paper, the main focus is on investigating the dilatational response of porous solids with matrix governed by Tresca's yield criterion, which is an even function of the stress deviator and involves both invariants. Such a study is also of interest in view of

* Corresponding author. Tel.: +1 850 833 9350; fax: +1 850 833 9366.
E-mail address: cazacu@reef.ufl.edu (O. Cazacu).

0020-7683/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijsolstr.2013.11.010

engineering applications, given that for certain fully-dense metallic materials Tresca yield criterion describes better the plastic response than von Mises criterion (e.g. annealed aluminum data reported by Vial-Edwards (1997); for a historical survey on early experimental work on mild steel tubes subjected to complex loading paths such as torsion and bending, torsion–compression, and torsion–tension tests the readers are referred to Michno and Findley (1976)). The maximum shear stress criterion of Tresca is also considered to be more physically-motivated because it is an isotropic form of the Schmid law describing slip at single-crystal level (for example, see Hughes, 1984). However, for most untextured metallic materials the yield locus is between that of Tresca's and von Mises (e.g. see Drucker, 1949) with von Mises criterion usually found more accurate, and this is why it is mostly applied.

Tresca's yield criterion postulates that in an isotropic metallic material the onset of plastic deformation occurs when the maximum shear stress over all planes in the material reaches a certain critical value. This criterion is generally represented as:

$$\varphi(\boldsymbol{\sigma}) = \sigma_T,$$

with

$$\varphi(\boldsymbol{\sigma}) = \max(|\sigma_1 - \sigma_2|,\ |\sigma_2 - \sigma_3|,\ |\sigma_1 - \sigma_3|). \tag{1}$$

In Eq. (1), σ_1, σ_2, and σ_3, are the principal values of the Cauchy stress tensor, $\boldsymbol{\sigma}$, and σ_T is the uniaxial yield in tension (see Lubliner, 2008). It may alternatively be written in terms of the invariants of the Cauchy stress deviator, σ', defined as: $\sigma' = \sigma - \sigma_m I$, with $\mathbf{I}$ being the 2nd order identity tensor and $\sigma_m = tr(\sigma)/3 = (\sigma_1 + \sigma_2 + \sigma_3)/3$ denoting the mean stress. Since isotropy dictates three fold symmetry of the yield surface, it is sufficient to give the expression of Tresca's criterion for stress states corresponding to the sextant $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$, i.e.

$$\sigma_1 - \sigma_3 = \sigma_T,$$

or, equivalently

$$2\sqrt{J_2}\cos\left(\frac{\pi}{6} - \beta\right) = \sigma_T. \tag{2}$$

In the above equation, β is the Lode angle satisfying $0 \leqslant \beta \leqslant \pi/3$ and whose cosine is: $\cos(3\beta) = \frac{J_3}{2}\left(\frac{3}{J_2}\right)^{\frac{3}{2}}$ while J_2 and J_3 are the second and third invariants of the stress deviator σ' (see also Malvern, 1969).

Since Tresca's criterion involves both the second and third-invariants of the Cauchy stress deviator (see Eq. (2)), yielding, plastic flow, and strength of a porous polycrystal containing randomly distributed spherical voids in a matrix governed by Tresca's criterion ought to incorporate all three invariants of stress. One of the major objectives of this paper is to derive such an yield criterion using rigorous limit-analysis theorems.

It is to be noted that a major difficulty in deriving such a criterion in closed-form is related to the calculation of the overall plastic dissipation. This is a direct consequence of Tresca's criterion involving a dependence on the third invariant of the stress deviator. Indeed, in contrast with the case when the matrix is described by the von Mises criterion, mathematical difficulties arise in the analysis because the expression of the local plastic dissipation depends on the sign of each of the principal values of the local strain rate tensor. It is shown that, despite these fresh difficulties associated with the calculation of the local plastic dissipation, all the integrals representing the overall plastic dissipation can be calculated analytically. The main result of this work, an explicit parametric representation of the yield surface for porous solids with randomly distributed spherical voids in a Tresca matrix is presented in Section 2.2. New and unexpected results are revealed; namely, that yielding of a porous solid with Tresca matrix should involve a very specific coupling between the mean stress and the third-invariant of the stress deviator. It is worth noting that this coupling is not postulated but it results from the analysis, which is based on rigorous upscaling techniques Moreover, axisymmetric finite-element (FE) cell calculations are conducted in order to generate numerical yield surfaces for a porous material with matrix's response described by Tresca yield criterion (Section 3). These numerical results also reveal the same coupling between the mean stress and the third invariant, which induces a lack of symmetry of the yield surface for stress-triaxialities different from 0 and $\pm\infty$. Currently, when calculating the effective response of porous solids with von Mises matrix, coupling between deviatoric and mean stress effects are neglected. In Section 4, we examine the consequences of adopting this approximation in the case of a Tresca matrix. The results obtained show that it amounts to erasing the specificities of the plastic flow of the matrix.

2. Derivation of the analytic yield criterion

2.1. Kinematic homogenization approach

We begin by briefly presenting the kinematic homogenization approach based on Hill–Mandel (Hill, 1967; Mandel, 1972) lemma that will be used to derive the closed-form expressions for the yield criterion of a porous solid with matrix obeying Tresca's criterion. Let Ω denote a representative volume element composed of a homogeneous rigid-plastic matrix and a traction-free void. If the matrix material is described by a convex yield function $\varphi(\boldsymbol{\sigma})$ in the stress space and an associated flow rule:

$$\mathbf{d} = \dot{\lambda}\frac{\partial\varphi}{\partial\boldsymbol{\sigma}}. \tag{3}$$

The plastic dissipation potential of the matrix is then defined as

$$\pi(\mathbf{d}) = \sup_{\boldsymbol{\sigma}\in C}(\boldsymbol{\sigma} : \mathbf{d}). \tag{4}$$

In the above equations, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{d} = (\nabla\mathbf{v} + \nabla\mathbf{v}^{\mathrm{T}})/\mathbf{2}$ is the strain rate tensor with $\mathbf{v}$ being the velocity field; ":" denotes the double-contracted product of the two tensors, $\dot{\lambda} \geqslant 0$ is the plastic multiplier rate while C denotes the convex domain delimited by the yield surface, i.e.

$$C = \{\boldsymbol{\sigma}|\varphi(\boldsymbol{\sigma}) \leqslant 0\}.$$

For uniform strain rate boundary conditions on $\partial\Omega$ such that

$$\mathbf{v} = \mathbf{D}\cdot\mathbf{x}, \quad \text{for any } \mathbf{x} \in \partial\Omega, \tag{5}$$

with $\mathbf{D}$ the overall strain rate tensor, Hill–Mandel (Hill, 1967; Mandel, 1972) lemma applies:

$$\langle\boldsymbol{\sigma} : \mathbf{d}\rangle_\Omega = \boldsymbol{\Sigma} : \mathbf{D}, \tag{6}$$

In the above equation, $\langle\ \rangle$ denotes the average value over the representative volume Ω, $\mathbf{D}$ is the overall strain rate tensor, and $\boldsymbol{\Sigma} = \langle\boldsymbol{\sigma}\rangle_\Omega$. Moreover, there exists a macroscopic strain rate potential $\Pi = \Pi(\mathbf{D})$, where

$$\Pi(\mathbf{D}) = \inf_{d\in K(\mathbf{D})}\langle\pi(\mathbf{d})\rangle_\Omega \text{ and } \Sigma = \frac{\partial\Pi(\mathbf{D})}{\partial\mathbf{D}} \tag{7}$$

(for more details, see Talbot and Willis, 1985). In Eq. (7), $K(\mathbf{D})$ denotes the set of incompressible velocity fields satisfying condition (5) (for more details, see for example, Michel and Suquet, 1992; Leblond, 2003; Garajeu and Suquet, 1997). This lemma will be further used to derive the plastic potential of a porous solid, with rigid-plastic matrix obeying Tresca yield function (i.e. $\varphi(\sigma)$ given by Eq. (1)). Since $\varphi(\sigma)$ is homogeneous of degree one in stresses,